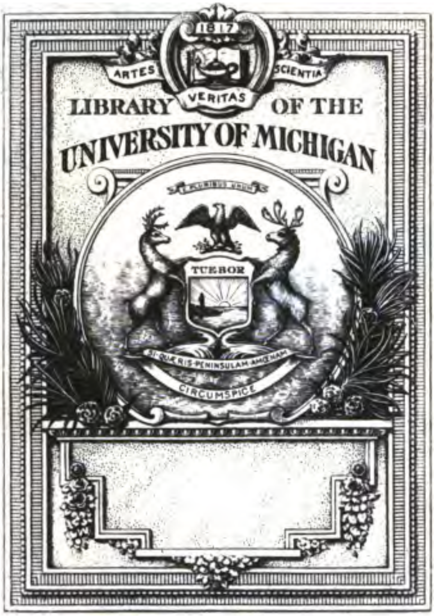
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# MENTAL AND PRACTICAL

# ARITHMETIC.

DESIGNED FOR THE USE OF ACADEMIES AND SCHOOLS.

BY CHARLES DAVIES.

AUTHOR OF ELEMENTS OF SURVEYING,
ELEMENTS OF DESCRIPTIVE GEOMETRY, SHADES SHADOWS AND PERSPECTIVE, ANALYTICAL GEOMETRY, AND DIFFERENTIAL AND INTEGRAL CALCULUS.

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# PREFACE.

A CORRECT and accurate knowledge of Arithmetic is one of the most important elements of a liberal or practical education. The public should, therefore, receive with indulgence every attempt that may be made to improve this department of instruction.

The Elementary Treatise, which is here presented, is an enlarged, and it is hoped, an improved edition of the Common School Arithmetic, published in 1833. The suggestions of several experienced teachers have been incorporated with the body of the work in its new form, and, indeed, nothing has been omitted which it was thought would give it value to those for whose use it is designed. It has been the intention to render the whole subject as plain as it is capable of being made, and at the same time, to treat it as concisely as possible.

The reasons for most of the rules are given. It was not, however, thought best to demonstrate the rules for the extraction of roots, nor that for finding the sum of a geometrical series.

The name, Compound Numbers, which has heretofore been given to all numbers in which the *kind* of unit is expressed, has been changed to that of Denominate Numbers. This change has not been made with any ambitious spirit of innovation, but because it is deemed an improvement

It is not easy to form an idea of what is meant by the term, Compound Number, and especially so, when we find it applied to such numbers as 3 pounds, 3 dollars, 3 shillings, &c. Why is 3 pounds a compound number any more than 3? If it be answered, that 3 pounds is composed or compounded of three single pounds, that does not remove the difficulty, for 3 is also composed of three units 1. Is it not then the better way to call the first a denominate number, and the other a simple number, as is done in § 45.

Mr. Hasler, in his Arithmetic, has called this class of numbers, Denominate Fractions.

In the present edition, the questions referring to each section are arranged directly after the section, which generally brings the question and answer on the same page. This alteration will, no doubt, be found convenient to teachers.

A Key has also been prepared, in which all the questions contained in the Arithmetic are resolved, and in such a manner, that the particular methods of solution can be fully understood. Many examples, not in the Arithmetic, have been imbodied in the Key, in order that the pupils may be exercised in questions not found in the books before them.

*Hartford, Connecticut, March*, 1838.

# CONTENTS.

# MENTAL ARITHMETIC.

## PART I.

1. How many eyes have you? How many ears have you? How many hands have you? How many thumbs have you? How many fingers have you on each hand? How many fingers have you on both hands?

If you have one apple and Charles gives you one, how many will you have? One and one are how many? If you have one apple and Charles gives you two, how many will you have? One and two are how many? If he gives you three, how many will you have? One and three are how many? If he gives you four, how many will you have? One and four are? One and five are? One and six are? One and seven are? One and eight are? One and nine are?

2. If you have two nuts in one hand and one in the other, how many have you in both? Two and one are how many? If you have two nuts in one hand and two in the other, how many have you in both? Two and two are how many? If you have two nuts in one hand and three in the other, how many have you in both? Two and three are how many? If you have two nuts in one hand and four in the other, how many have you in both? Two and four are? Two and five are? Two and six are? Two and seven are? Two and eight are? Two and nine are?

3. If you have three peaches in one basket and one in another, how many have you in both? Three and one are? If you have three in one basket and two in the other, how many? Three and two are? If you have three in one basket and three in the other, how many? Three and three are? Three and four are? Three and five

are? Three and six are? Three and seven are? Three and eight are? Three and nine are?

4. If you have four oranges and buy one, how many will you have? Four and one are how many? If you buy two, how many will you have? Four and two are how many? If you buy three, how many will you have? Four and three are how many? Four and four are? Four and five are? Four and six are? Four and seven are? Four and eight are? Four and nine are?

5. If you have five pears and mother gives you one, how many will you have? Five and one are how many? If she gives you two, how many will you have? Five and two are how many? If she gives you three, how many? Five and three are? Five and four are? Five and five are? Five and six are? Five and seven are? Five and eight are? Five and nine are?

6. If you have six marbles and James gives you one, how many will you have? Six and one are how many? If James gives you two, how many will you have? Six and two are how many? If he gives you three, how many will you have? Six and three are how many? Six and four are? Six and five are? Six and six are? Six and seven are? Six and eight are? Six and nine are?

7. If you have seven figs and buy one, how many will you have? Seven and one are how many? If you buy two, how many will you have? Seven and two are how many? If you buy three, how many will you have? Seven and three are how many? Seven and four are? Seven and five are? Seven and six are? Seven and seven are? Seven and eight are? Seven and nine are?

8. If you have eight pencils and George gives you one, how many will you have? Eight and one are how many? If George gives you two, how many will you have? Eight and two are how many? If he gives you three, how many will you have? Eight and three are how many? Eight and four are? Eight and five are? Eight and six are? Eight and seven are? Eight and eight are? Eight and nine are?

9. If you have nine lemons and buy one, how many will you have? Nine and one are how many? If you

buy two, how many will you have? Nine and two are? If you buy three, how many will you have? Nine and three are? Nine and four are? Nine and five are? Nine and six are? Nine and seven are? Nine and eight are? Nine and nine are?

10. John has three apples, Charles five, and James four, how many have all three? Three, five and four are how many.

11. John bought six cents worth of raisins, Charles bought four cents worth of figs, and James three cents worth of apples; how much did they all pay? Six and four and three are how many?

12. One box contains eight cents, one four, and one seven; how many are there in the three? Eight and four and seven are how many?

13. Charles gave four cents for a pencil, three cents for an orange, and fifteen cents for a knife; how many cents did he pay for the whole?

14. If you give twelve cents for a spelling-book, and eight cents for an ink-stand; what do they come to?

15. A man bought twelve pounds of tea for eight dollars, six pounds of coffee for one dollar, and five yards of cloth for six dollars; what did the whole come to?

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## PART II.

1 If John has one apple and gives it to Charles, how many will he have left? One from one and what remains? If he has two apples and gives one to Charles, how many will he have left? One from two and what remains? If he has three apples and gives one to Charles, how many will he have left? One from three and what remains? One from four and what remains? One from five and what remains? One from six and what remains? One from seven and what remains? One from eight and what remains? One from nine and what remains? One from ten and what remains?

2. If John has two apples and gives two to Charles, how many will he have left? Two from two and what remains?

1*

If he has three and gives two to Charles, how many will he have left? Two from three and what remains? If he has four and gives two to Charles, how many will he have left? Two from four and what remains? Two from five and what remains? Two from six and what remains? Two from seven and what remains? Two from eight and what remains? Two from nine and what remains? Two from ten and what remains?

3. If John has three oranges and gives three to Charles, how many will he have left? Three from three and what remains? If he has four and gives three to Charles, how many will he have left? Three from four and what remains? If he has five and gives three to Charles, how many will he have left? Three from five and what remains? Three from six and what remains? Three from seven and what remains? Three from eight and what remains? Three from nine and what remains? Three from ten and what remains?

4. If John has four nuts and gives four to Charles, how many will he have left? Four from four and what remains? If he has five and gives four to Charles, how many will he have left? Four from five and what remains? Four from six and what remains? Four from seven and what remains? Four from eight and what remains? Four from nine and what remains? Four from ten and what remains?

5. If John has five peaches and gives five to Charles, how many will he have left? Five from five and what remains? If he has six and gives five to Charles, how many will he have left? Five from six and what remains? Five from seven and what remains? Five from eight and what remains? Five from nine and what remains? Five from ten and what remains?

6. If John has six figs and gives six to Charles, how many will he have left? Six from six and what remains? If he has seven and gives six to Charles, how many will he have left? Six from seven and what remains? Six from eight and what remains? Six from nine and what remains? Six from ten and what remains?

7. If James has seven lemons and gives seven to

Charles, how many will he have left? Seven from seven and what remains? If he has eight and gives seven to Charles, how many will he have left? Seven from eight and what remains? Seven from nine and what remains? Seven from ten and what remains?

8. If James has eight pencils and gives eight to Charles, how many will he have left? Eight from eight and what remains? If he has nine and gives eight to Charles, how many will he have left? Eight from nine and what remains? Eight from ten and what remains?

9. If James has nine marbles and gives nine to Charles, how many will he have left? Nine from nine and what remains? Nine from ten and what remains?

10. If he has ten and gives nine away, how many will he have left? Nine from ten and what remains?

11. Charles has eight marbles and loses three, how many has he left? If he has twelve and loses four, how many will he have left? If John gives twelve cents for a knife and nine cents for a pencil, how much more does his knife cost than the pencil?

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## PART III.

1. Charles buys one pencil for one cent, how much does it cost? One times one is how many? If he buys two pencils for one cent each, how much do they cost? One times two is how many? If he buys three pencils for one cent each, how much do they cost? One times three is how many? One times four is how many? One times five is how many? One times six is? One times seven is? One times eight is? One times nine is? One times ten is?

2. If Charles buys one marble for two cents, how much does it cost? Two times one are how many? If he buys two marbles for two cents each, how much do they cost? Two times two are how many? If he buys three marbles for two cents each, how much do they cost him? Two times three are how many? Two times four are? Two

times five are? Two times six are? Two times seven are? Two times eight are? Two times nine are? Two times ten are?

3. If Charles buys one orange for three cents, what does it cost him? Three times one are how many? If he buys two oranges for three cents each, how much do they cost him? Three times two are how many? If he buys three oranges for three cents each, what do they cost him? Three times three are how many? Three times four are? Three times five are? Three times six are? Three times seven are? Three times eight eight are? Three times nine are? Three times ten are?

4. If Charles buys one book for four cents, what does it cost him? Four times one are how many? If he buys two books for four cents each, how much will they cost him? Four times two are how many? If he buys three books for four cents each, how much will they cost him? Four times three are how many? If he buys four at the same price, what will they cost? Four times four are? Four times five are? Four times six are? Four times seven are? Four times eight are? Four times nine are? Four times ten are?

5. If Charles buys one pocket-handkerchief for five cents, what does it cost him? Five times one are how many? If he buys two for five cents each, how much will they cost him? Five times two are how many? If he buys three for five cents each, how much do they cost him? Five times three are how many? Five times four are how many? Five times five are? Five times six are? Five times seven are? Five times eight are? Five times nine are? Five times ten are?

6. If Charles buys one whistle for six cents, how much does it cost him? Six times one are how many? If he buys two whistles for six cents each, how much do they cost him? Six times two are how many? If he buys three whistles for six cents each, how much do they cost him? Six times three are how many? Six times four are how many? Six times five are how many? Six times six are? Six times seven are? Six times eight are? Six times nine are? Six times ten are?

7. If Charles buys one ball for seven cents, how much does it cost him? Seven times one are how many? If he buys two balls for seven cents each, how much do they cost him? Seven times two are how many? If he buys three at the same price, how much will they cost him? Seven times three are how many? Seven times four are how many? Seven times five are? Seven times six are? Seven times seven are? Seven times eight are? Seven times nine are? Seven times ten are?

8. If John's father buys one yard of cloth for eight dollars, how much will it cost him? Eight times one are how many? If he buys two yards and gives eight dollars for each, how much will they cost him? Eight times two are how many? If he buys three yards for eight dollars each yard, how much will they cost him? Eight times three are how many? Eight times four are? Eight times five are? Eight times six are? Eight times seven are? Eight times eight are? Eight times nine are? Eight times ten are?

9. If Charles buys one knife for nine cents, how much will it cost him; Nine times one are how many? If he buys two knives for nine cents each, how much will they cost him? Nine times two are how many? If he buys three knives for nine cents each, how much will they cost him? Nine times three are how many? Nine times four are? Nine times five are? Nine times six are? Nine times seven are? Nine times eight are? Nine times nine are? Nine times ten are?

10. If Charles buys one hammer for ten cents, how much will it cost him? Ten times one are how many? If he buys two hammers for ten cents each, how much will they cost him? Ten times two are how many? If he buys three hammers at the same price, how much will they cost him? Ten times three are how many? Ten times four are? Ten times five are? Ten times six are? Ten times seven are? Ten times eight are? Ten times nine are? Ten times ten are?

11. If I give three cents for one yard of tape, what will nine yards cost? How many cents will buy eight marbles, if the marbles cost five cents a piece? If one

bushel of wheat cost one dollar, what will twenty bushels cost? If one bushel of clover seed cost nine dollars, what will eight bushels cost? What will nine bushels cost?

12. If four apples will buy one orange, how many apples will buy two oranges? How many will buy three? How many will buy four? How many will buy five?

13. A man sold seven sheep for four dollars a piece, how much did they come to? He sold eight more at three dollars a piece, how much did they come to? He then sold seven for five dollars a piece, how much did they come to?

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## PART IV.

1. If two apples be equally divided between two boys, how many will each have? Two in two how many times? If four apples be equally divided between two boys, how many will each have? Two in four how many times? If six apples be equally divided between two boys, how many will each have? Two in six how many times? If eight be divided between them, how many will each have? Two in eight how many times? Two in ten how many times? Two in twelve how many times? Two in fourteen how many times? Two in sixteen how many times? Two in eighteen how many times? Two in twenty how many times?

2. If three apples be equally divided between three boys, how many will each have? Three in three how many times? If nine apples be equally divided among three boys, how many will each have? Three in nine how many times? If twelve apples be equally divided, how many will each have? Three in twelve how many times? Three in fifteen how many times? Three in eighteen how many times? Three in twenty-one how many times? Three in twenty-four how many times? Three in twenty-seven how many times? Three in thirty how many times?

3. If four apples be equally divided among four boys, how many will each have? Four in four how many times? If eight apples be equally divided among four boys, how many will each have? Four in eight how many times? If twelve apples be thus divided, how many will each have? Four in twelve how many times? Four in sixteen how many times? Four in twenty how many times? Four in twenty-four how many times? Four in twenty-eight how many times? Four in thirty-two how many times? Four in thirty-six how many times? Four in forty how many times?

4. If five apples be equally divided among five boys, how many will each have? Five in five how many times? If ten apples be equally divided among five boys, how many will each have? Five in ten how many times? Five in fifteen how many times? Five in twenty how many times? Five in twenty-five how many times? Five in thirty how many times? Five in thirty-five how many times? Five in forty how many times? Five in forty-five how many times? Five in fifty how many times?

5. If six apples be equally divided among six boys, how many will each have? Six in six how many times? If twelve apples be equally divided among six boys, how many will each have? Six in twelve how many times? Six in eighteen how many times? Six in twenty-four how many times? Six in thirty how many times? Six in thirty-six how many times? Six in forty-two how many times? Six in forty-eight how many times? Six in fifty-four how many times? Six in sixty how many times?

6. If seven apples be equally divided among seven boys, how many will each have? Seven in seven how many times? If fourteen apples be equally divided among seven boys, how many will each have? Seven in fourteen how many times? Seven in twenty-one how many times? Seven in twenty-eight how many times? Seven in thirty-five how many times? Seven in forty-two how many times? Seven in forty-nine how many times? Seven in fifty-six how many times? Seven in sixty-three how many times? Seven in seventy how many times?

7. If eight apples be equally divided among eight boys, how many will each boy have? Eight in eight how many times? If sixteen apples be equally divided among eight boys, how many will each have? Eight in sixteen how many times? Eight in twenty-four how many times? Eight in thirty-two how many times? Eight in forty how many times? Eight in forty-eight how many times? Eight in fifty-six how many times? Eight in sixty-four how many times? Eight in seventy-two how many times? Eight in eighty how many times?

8. If nine apples be equally divided among nine boys, how many will each have? Nine in nine how many times? If eighteen apples be equally divided among nine boys, how many will each have? Nine in eighteen how many times? Nine in twenty-seven how many times? Nine in thirty-six, how many times? Nine in forty-five how many times? Nine in fifty-four how many times? Nine in sixty-three how many times? Nine in seventy-two, how many times? Nine in eighty-one how many times? Nine in ninety how many times?

9. If ten apples be equally divided among ten boys, how many will each have? Ten in ten how many times? If twenty apples be equally divided among ten boys how many will each have? Ten in twenty how many times? Ten in thirty how many times? Ten in forty how many times? Ten in fifty how many times? Ten in sixty how many times? Ten in seventy how many times? Ten in eighty how many times? Ten in ninety how many times? Ten in one hundred how many times?

10. Thirty-six cents are given to six boys, how many does each boy get? Twelve dollars are given to three boys, how much to each? Twenty dollars are given to four boys, how much to each?

11. If six lead pencils cost fifty-four cents, how much are they apiece? If five oranges cost forty cents, how much are they apiece? If nine yards of cloth cost twenty-seven dollars, how much does it cost per yard? If nine yards of riband cost eighty-one cents, how much is it a yard?

## PART V.

1. If a single thing, as one apple, be divided into two equal parts, each part is called a *half*.

How many halves in one thing? How many halves in two? How many in three? How many in four? How many in five? How many in six? How many in seven? How many in eight? How many in nine? How many in ten?

2. If a single thing, as one apple, be divided into three equal parts, each part is called a *third*.

How many thirds are there in one thing? How many thirds in two things? How many thirds in three things? How many thirds in four things? How many in five? How many in six? How many in seven? How many in eight? How many in nine? How many in ten?

3. If a single thing be divided into four equal parts, each part is called a *fourth*.

How many fourths are there in one thing? How many in two things? How many in three? How many in four? How many in five? How many in six? How many in seven? How many in eight? How many in nine? How many in ten?

4. If a single thing be divided into five equal parts, each part is called a *fifth*.

How many fifths are there in one thing? How many in two? How many in three? How many in four? How many in five? How many in six? How many in seven? How many in eight? How many in nine? How many in ten?

5. If a single thing be divided into six equal parts, each part is called a *sixth*.

How many sixths are there in one thing? How many sixths are there in two things? How many are there in three? How many in four? How many in five? How many in six? How many in seven? How many in eight? How many in nine? How many in ten?

6. If a single thing be divided into seven equal parts, each part is called a *seventh*.

How many sevenths are there in a single thing? How many sevenths are there in two things? How many sevenths are there in three things? How many in four? How many in five? How many in six? How many in seven? How many in eight? How many in nine? How many in ten?

7. If a single thing be divided into eight equal parts, each part is called an *eighth*.

How many eighths are there in one thing? How many in two things? How many in three? How many in four? How many in five? How many in six? How many in seven? How many in eight? How many in nine? How many in ten?

8. If a single thing be divided into nine equal parts, each part is called a *ninth*.

How many ninths are there in one thing? How many ninths are there in two things? How many ninths are there in three? How many in four? How many in five? How many in six? How many in seven? How many in eight? How many in nine? How many in ten?

9. If a single thing be divided into ten equal parts, each part is called a *tenth*.

How many tenths are there in one thing? How many tenths are there in two things? How many in three things? How many in four? How many in five! How many in six? How many in seven? How many in eight? How many in nine? How many in ten?

10. How many halves make five? How many halves make nine? How many halves make fifteen? How many halves make eighteen? How many halves make twenty-five? How many halves make twenty-two?

11. How many things in three-thirds? *Answer*. One. How many things in four-thirds? *Ans*. One and one-third. How many in five-thirds? *Ans*. One and two-thirds. How many in six-thirds? How many in seven-thirds? How many in eight-thirds? How many in nine-thirds? How many in ten-thirds.

12. How many things are there in four-fourths? *Ans*. One. How many things are there in five-fourths? *Ans*. One and one-fourth. How many things in six-fourths?

How many in seven-fourths? How many in eight-fourths? How many in nine-fourths? How many in ten-fourths? How many in eleven-fourths? How many in twelve-fourths?

13. How many things are there in five-fifths? *Ans.* One. How many things are there in six-fifths? *Ans.* One and one-fifth. How many in seven-fifths? How many in eight-fifths? How many in nine-fifths? How many in ten-fifths? How many in eleven-fifths? How many in twelve-fifths? How many in thirteen-fifths? How many in fourteen-fifths? How many in fifteen-fifths?

14. How many things are there in six-sixths? *Ans.* One. How many are there in seven-sixths? *Ans.* One and one-sixth? How many in eight-sixths? How many in nine-sixths? How many in ten-sixths? How many in eleven-sixths? How many in twelve-sixths?

15. How many things are there in seven-sevenths? *Ans.* One. How many things are there in eight-sevenths? *Ans.* One and one-seventh. How many in nine-sevenths? How many in ten-sevenths? How many in eleven-sevenths? How many in twelve-sevenths? How many in thirteen-sevenths? How many in fourteen-sevenths?

16. How many things are there in eight-eighths. *Ans.* One. How many things are there in nine-eighths? *Ans.* One and one-eighth. How many in ten eighths? How many in eleven-eighths? How many in twelve-eighths? How many in thirteen-eighths? How many in fourteen-eighths? How many in fifteen-eighths? How many in sixteen-eighths?

17. How many things are there in nine-ninths? *Ans.* One. How many things are there in ten-ninths? *Ans.* One and one-ninth. How many things are there in eleven-ninths? How many in twelve-ninths? How many in thirteen-ninths? How many in fourteen-ninths? How many in fifteen-ninths? How many in sixteen-ninths? How many in seventeen-ninths? How many in eighteen-ninths?

18. How many things are there in ten-tenths? *Ans.* One. How many things are there in eleven-tenths? *Ans.* One and one-tenth? How many in twelve-tenths? How

many in thirteen-tenths? How many in fourteen-tenths? How many in fifteen-tenths? How many in sixteen-tenths? How many in seventeen-tenths? How many in eighteen tenths? How many in nineteen-tenths? How many in twenty-tenths?

19. If you pay twelve dollars for two weeks board, how much is that for one week? If you pay three dollars a week for board, how much will you pay for five weeks? If you travel four miles an hour, how far can you go in ten hours? If you travel sixty miles in four hours, how far will you travel in one hour?

20. If you pay eight dollars for a piece of cloth containing sixteen yards, how much do you pay for a yard? If you pay twenty dollars for a piece of cloth containing five yards, how much do you pay for a yard?

21. If you pay thirty cents for ten oranges, how much is that apiece? If you pay three cents apiece for twelve oranges, how much will they come to?

22. If you pay four cents for a knife, six cents for a jews-harp, and eight cents for a dozen marbles, how much do they all cost?

23. If you pay one dollar for a pair of shoes, three dollars for a hat, and six dollars for a coat, what do they all cost? If you pay twelve cents for tape, nine cents for riband, and eight cents for pins, what do they all cost?

24. If you buy a cow for eight dollars, three sheep at two dollars apiece, and a horse for one hundred dollars, what do they all come to?

25. A yoke of oxen costs seventy-five dollars, and a horse seventy-five dollars, what do they come to?

26. My coat costs me twenty-eight dollars, my pantaloons twelve dollars, and my vest five dollars, what does the whole suit cost me?

27. My hat costs eight dollars, my boots seven dollars, and my over-coat twenty-nine dollars, what do they all cost?

28. A man buys a pair of horses for one hundred dollars, and three more horses for sixty dollars each, what do they all cost him?

# ARITHMETIC.

## NUMERATION AND NOTATION.

§ 1. A single thing is called - - - *One*,
One and one more are called - - *Two*,
Two and one more are called - - *Three*,
Three and one more are called - - *Four*,
Four and one more are called - - *Five*,
Five and one more are called - - *Six*,
Six and one more are called - - *Seven*,
Seven and one more are called - - *Eight*,
Eight and one more are called - - *Nine*,
Nine and one more are called - - *Ten*,
&c. &c. &c. &c. &c.

Each word, *one*, *two*, *three*, *four*, *five*, *six*, &c., points out how many things are spoken of. These words are called NUMBERS. Hence, NUMBERS *are the expressions for several things of the same kind.*

*Questions.* What is a single thing called? One and one? Two and one? Three and one? Four and one? Five and one? Six and one? Seven and one? &c. What are Numbers?

§ 2. The *unit* of a number is one of the equal things which the number expresses. Thus, if the number be six apples, one apple is the unit; if it be five pounds of tea, one pound of tea is the unit; if it be ten feet of length, one foot is the unit; if it be four hours of time, one hour is the unit.

Q. What is the unit of a number? What is the unit of the number six apples? Of the number five pounds of tea? Of the number ten feet? Of the number four hours?

§ 3. Arithmetic treats of numbers. Numbers are expressed by certain characters, called figures. There are ten of these characters. They are

| | | |
|---|---|---|
| 0 | which is called a cipher, or | Naught, |
| 1 | - - - - | One, |
| 2 | - - - - | Two, |
| 3 | - - - - | Three, |
| 4 | - - - - | Four, |
| 5 | - - - - | Five, |
| 6 | - - - - | Six, |
| 7 | - - - - | Seven, |
| 8 | - - - - | Eight, |
| 9 | - - - - | Nine. |

Q. Of what does arithmetic treat? How are numbers expressed? How many figures are there? Name them.

§ 4. The character 0 is used to denote the absence of a thing. As, if we wish to express by figures that there are no apples in a basket, we write, the number of apples in the basket is 0. The nine other figures are called *significant figures, or Digits.*

| | | | |
|---|---|---|---|
| 1 | expresses a single thing, or a *unit* of a number. | | |
| 2 | - | two things of the same kind, | or two units. |
| 3 | - | three things - - | or three units. |
| 4 | - | four things - - | or four units. |
| 5 | - | five things - - | or five units. |
| 6 | - | six things - - | or six units. |
| 7 | - | seven things - - | or seven units. |
| 8 | - | eight things - - | or eight units. |
| 9 | - | nine things - - | or nine units. |

Q. What does 0 express? What are the nine other figures called? How many things does 1 express? How many things does 2 express? How many units in 3? In 4? In 5? In 6? In 7? In 8? In 9?

§ 5. If we wish to express the number *ten*, we have no separate character for it. We must *combine* the characters already known. This we do by writing 0 on the right hand of the 1; thus, 10, which is read *ten*.

This 10 is equal to *ten* of the units expressed by 1. It is, however, but a *single ten*, and in this sense may be regarded as a *unit*, the value of which is *ten times greater* than the unit expressed by 1. It is called a unit of the *second order.*

Q. Have we a separate character for ten? How do we express ten? To how many units 1 is it equal? May we consider it a single unit? Of what order?

§ 6. When two figures are written by the side of each other, the one on the right is called the *place of units*, the other, the *place of tens*, or *units of the second order;* and *each unit of the second order is equal to ten units of the first order.*

When units simply are named, *units of the first order are always meant.*

| | |
|---|---|
| Two tens, or twenty, are written - | 20 |
| Three tens, or thirty, - - - - | 30 |
| Four tens, or forty, - - - - | 40 |
| Five tens, or fifty, - - - - | 50 |
| Six tens, or sixty, - - - - | 60 |
| Seven tens, or seventy, - - - | 70 |
| Eight tens, or eighty, - - - - | 80 |
| Nine tens, or ninety, - - - - | 90 |

The intermediate numbers between 10 and 20, between 20 and 30, &c., may be readily expressed by considering the tens and units of which they are composed. For example, the number twelve is made up of one ten and two units. It must therefore be written by setting 1 in the place of tens, and 2 in the place of units; thus, - 12

| | |
|---|---|
| Eighteen has 1 ten and 8 units, and is written - - | 18 |
| Twenty-five has 2 tens and 5 units, and is written - | 25 |
| Thirty-seven has 3 tens and 7 units, and is written - | 37 |
| Fifty-four has 5 tens and 4 units, and is written - - | 54 |
| Eighty-nine has 8 tens and 9 units, and is written - | 89 |
| Ninety-nine has 9 tens and 9 units, and is written - | 99 |

Hence, any number greater than nine and less than one hundred, may be expressed by two figures.

Q. When two figures are written by the side of each other, what is the place on the right called? The place on the left? When units simply are named, what units are meant? How many units of the second order in 20? In 30? In 40? In 50? In 60? In 70? In 80? In 90? Of what is the number 12 made up? Also, 18, 25, 37, 54, 89, 99? What numbers may be expressed by a single figure? What numbers may be expressed by two figures?

§ 7. In order to express one hundred, or *ten units of the second order*, we have to form a new combination.

It is done thus, - - - - - 100

by writing two ciphers on the right of 1. This number is read, one hundred. Now this one hundred expresses 10 *units of the second order*, or *one hundred units of the first order*. But the one hundred is but *an individual hundred*, and in this light may be regarded as a unit of the *third order*.

We can now express any number less than one thousand.

For example, in the number three hundred and seventy-five, there are 3 hundreds, 7 tens, and 5 units. We are, therefore, to express 3 units of the 3d order, 7 units of the second order, and 5 of the 1st.

| huns. | tens. | units. |
|---|---|---|
| 3 | 7 | 5 |

Hence, we write - - - - - - - 375

and we read from the right, *units, tens, hundreds*.

In the number eight hundred and ninety-nine, there are 8 units of the 3d order, and 9 of the 2d, and 9 of the 1st.

| huns. | tens. | units. |
|---|---|---|
| 8 | 9 | 9 |

It is written - - - - - - - 899

and read, *units, tens, hundreds*.

In the number four hundred and six, there are 4 units of the 3d order, 0 of the 2d, and 6 of the 1st.

| huns. | tens. | units. |
|---|---|---|
| 4 | 0 | 6 |

It is written - - - - - - - 406

and in a similar manner we may express, by three figures, any number greater than ninety-nine and less than one thousand.

Q. How do you express one hundred? To how many units of the 2d order is it equal? To how many of the 1st order? May it be considered a single unit? Of what order is it? How many units of the 3d order in 200? In 300? In 400? In 500? In 600? In 700? In 800? In 900? Of what is the number 375 composed? The number 406? What numbers may be expressed by three figures?

§ 8. To express *ten* units of the 3d order, or one thousand, we form a new combination by writing three ciphers on the right of 1; thus, - - - 1000

Now, although this thousand expresses one thousand units of the 1st order, it is, nevertheless, but *one single thousand*, and may be regarded as a unit of the 4th order.

Proceeding in this way, we may place as many figures in a row as we please. When so placed, we conclude:

1st. *That the same figure has different values according to the place which it occupies.*

2d. *That counting from the right hand towards the left, the first is the place of units; the second, the place of tens; the third, the place of hundreds; the fourth, the place of thousands; &c.*

3d. *That ten units of the first place are equal to one unit of the second place; that ten units of the second place are equal to one unit of the third place; that ten units of the third place are equal to one unit of the fourth place; and so on, for places farther to the left.*

Q. To what are ten units of the 3d order equal? How do you express them? May this be considered a single unit? Of what order? May any number of figures be written in a row? When so placed has the same figure different values? On what does the value of the same figure depend? What is the first place on the right called? What is the second called? What is the third called? What is the fourth called? What are ten units of the first place equal to? What are ten units of the second place equal to? To what are ten units of the third place equal.

§ **9**. Expressing or writing numbers by figures, is called Notation. Reading the order of their places, correctly, when written, is called Numeration.

Q. What is Notation? What is Numeration? Which way do you numerate?

1. Write three tens. *Ans.* 30.
2. Write one hundred and fifty. *Ans.* 150.
3. Write twelve tens. *Ans.* 120.
4. Write 4 units of the first order, 5 of the 2d, 6 of the 3d, and 8 of the 4th. *Ans.* 8654.
5. Write 9 units of the 5th order, none of the 4th, 8 of the 3d, 7 of the 2d, and 6 of the 1st. *Ans.* 90876.
6. Write 1 unit of the 6th order, 5 of the 5th, 4 of the 4th, 9 of the 3d, 7 of the 2d, and none of the 1st. *Ans.* 154970.

## NUMERATION TABLE.*

| 6th Period or Period of Quadrillions. | 5th Period or Period of Trillions. | 4th Period or Period of Billions. | 3d Period or Period of Millions. | 2d Period or Period of Thousands. | 1st Period or Period of Units. |
|---|---|---|---|---|---|
| Hundreds of Quadrillions. Tens of Quadrillions Quadrillions | Hundreds of Trillions Tens of Trillions Trillions | Hundreds of Billions Tens of Billions Billions | Hundreds of Millions Tens of Millions Millions | Hundreds of Thousands Tens of Thousands Thousands | Hundreds Tens Units |
| . | . | . | . | . | 6 |
| . | . | . | . | . | 75 |
| . | . | . | . | . | 879 |
| . | . | . | . | 6, | 023 |
| . | . | . | . | 82, | 301 |
| . | . | . | . | 123, | 087 |
| . | . | . | 7, | 628, | 735 |
| . | . | . | 43, | 210, | 460 |
| . | . | . | 548, | 721, | 087 |
| . | . | 6, | 245, | 289, | 421 |
| . | . | 72, | 549, | 136, | 822 |
| . | . | 894, | 602, | 043, | 288 |
| . | 7, | 641, | 248, | 907, | 456 |
| . | 84, | 912, | 876, | 410, | 285 |
| . | 912, | 761, | 257, | 327, | 826 |
| 6, | 407, | 212, | 936, | 876, | 541 |
| 57, | 289, | 678, | 541, | 297, | 313 |
| 920, | 323, | 842, | 768, | 319, | 675 |

The words at the head of the numeration table, *units*, *tens*, *hundreds*, &c., are equally applicable to all numbers, and must be committed to memory, after which, the pupil may read the Table.

* NOTE.—This Table is formed according to the French method of numeration. The English method gives six places to thousands, &c.

To make the reading of figures easy, they are often separated into periods of three figures each, counting from the right hand.

EXAMPLES IN EXPRESSING NUMBERS BY FIGURES.

1. Write four in figures, *Ans.* 4.
2. Write four tens or forty. *Ans.* 40.
3. Write four hundred. *Ans.* 400.
4. Write four thousand. *Ans.* 4000.
5. Write forty thousand *Ans.* 40,000.
6. Write four hundred thousand. *Ans.* 400,000.
7. Write four millions. *Ans.* 4,000,000.

These examples show us very clearly that the same significant figure will have different values acccording to the place which it occupies.

8. Write seven. Write six units of the 2d order. Write nine units of the 3d order. Write six units of the 4th order. Write eight units of the 2d order. Write one unit of the third order. Write nine units of the 6th order. Write two units of the 8th order.

9. Write six hundred and seventy-nine. *Ans.* 679.
10. Write six thousand and twenty-one.
11. Write two thousand and forty.
12. Write one hundred and five thousand and seven.
13. Write three billions.
14. Write ninety-five quadrillions.
15. Write one hundred and six trillions, four thousand and two.
16. Write fifty-nine trillions, fifty-nine billions, fifty-nine millions, fifty-nine thousands, fifty-nine hundreds, and fifty-nine.
17. Write eleven thousand, eleven hundred and eleven.
18. Write nine billions and sixty-five.
19. Write three hundred and four trillions, one million, three hundred and twenty-one thousand, nine hundred and forty-one.

§ **10**. There is yet another method of expressing numbers, called the Roman. In this method the numbers are represented by letters. The letter I stands for *one;* V, five; X, ten; L, fifty; C, one hundred; D, five hundred, &c.

ROMAN TABLE

| | | | |
|---|---|---|---|
| I. | One | LX. | Sixty |
| II. | Two | LXX. | Seventy |
| III. | Three | LXXX. | Eighty |
| IV. | Four | XC. | Ninety |
| V. | Five | C. | One hundred |
| VI. | Six | CC. | Two hundred |
| VII. | Seven | CCC. | Three hundred |
| VIII. | Eight | CCCC. | Four hundred |
| IX. | Nine | D. | Five hundred |
| X. | Ten | DC. | Six hundred |
| XX. | Twenty | DCC. | Seven hundred |
| XXX. | Thirty | DCCC. | Eight hundred |
| XL. | Forty | DCCCC. | Nine hundred |
| L. | Fifty | M. | One thousand. |

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## ADDITION OF SIMPLE NUMBERS.

§ **11**. John has three apples and Charles has two; how many apples have they between them.

Every boy will answer, five.

Here a single apple is the unit, and the number five contains as many units as the two numbers three and two. The operation by which this result is obtained is called *addition*. Hence,

ADDITION *is the uniting together of several numbers, in such a way, that all the units which they contain may be expressed by a single number.*

Such single number is called the *sum* or *sum total* of the other numbers. Thus, 5 is the sum of the apples possessed by John and Charles.

What is the sum of 2 and 4? of 3 and 5? of 6 and 3? of 4, 3 and 1? of 2, 3 and 4? of 1, 2, 3 and 4? of 5 and 7? How many units in 4 and 6? How many units in 9 and 4?

Q. What is addition? What is the single number called which expresses all the units of the numbers added? What is the sum of 2 and 4? What is six called?

### OF THE SIGNS.

§ **12**. The sign +, is called *plus*, which signifies more. When placed between two numbers it denotes that they are to be added together. Thus, 3+2 denotes that 3 and 2 are to be added together.

The sign =, is called the sign of equality. When placed between two numbers it denotes that they are equal to each other.

Thus, 3+2=5. When the numbers are small we generally read them, by saying, 3 and 2 are 5.

**Q. What is the sign of addition? What is it called? What does it signify? When placed between two numbers what does it express? Express the sign of equality. When placed between two numbers what does it show? Give an example.**

§ **13.** Before adding large numbers the pupil should be able to add, in his mind, any two of the ten figures. Let him commit to memory the following table, which is read, two and 0 are two; two and one are three; two and two are four, &c.

## ADDITION TABLE.

| | | | |
|---|---|---|---|
| 2+0= 2 | 3+0= 3 | 4+0= 4 | 5+0= 5 |
| 2+1= 3 | 3+1= 4 | 4+1= 5 | 5+1= 6 |
| 2+2= 4 | 3+2= 5 | 4+2= 6 | 5+2= 7 |
| 2+3= 5 | 3+3= 6 | 4+3= 7 | 5+3= 8 |
| 2+4= 6 | 3+4= 7 | 4+4= 8 | 5+4= 9 |
| 2+5= 7 | 3+5= 8 | 4+5= 9 | 5+5=10 |
| 2+6= 8 | 3+6= 9 | 4+6=10 | 5+6=11 |
| 2+7= 9 | 3+7=10 | 4+7=11 | 5+7=12 |
| 2+8=10 | 3+8=11 | 4+8=12 | 5+8=13 |
| 2+9=11 | 3+9=12 | 4+9=13 | 5+9=14 |
| 6+0= 6 | 7+0= 7 | 8+0= 8 | 9+0= 9 |
| 6+1= 7 | 7+1= 8 | 8+1= 9 | 9+1=10 |
| 6+2= 8 | 7+2= 9 | 8+2=10 | 9+2=11 |
| 6+3= 9 | 7+3=10 | 8+3=11 | 9+3=12 |
| 6+4=10 | 7+4=11 | 8+4=12 | 9+4=13 |
| 6+5=11 | 7+5=12 | 8+5=13 | 9+5=14 |
| 6+6=12 | 7+6=13 | 8+6=14 | 9+6=15 |
| 6+7=13 | 7+7=14 | 8+7=15 | 9+7=16 |
| 6+8=14 | 7+8=15 | 8+8=16 | 9+8=17 |
| 6+9=15 | 7+9=16 | 8+9=17 | 9+9=18 |

$2 + 3 =$ how many?
$1 + 2 + 4 =$ how many?
$2 + 3 + 5 + 1 =$ how many?
$6 + 7 + 2 + 3 =$ how many?
$1 + 6 + 7 + 2 + 3 =$ how many?
$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 =$ how many?

1. What is the sum of 3 and 3 tens? *Ans.* 33
2. What is the sum of 8 tens and 9? *Ans.* 89.
3. What is the sum of 4, 5, and 4 tens? *Ans.* 49.
4. What is the sum of 1, 2, 3, 4, and 9 tens? *Ans.* 100.
5. What is the sum of 1, 2, 3, 4, 5, and 6 tens? *Ans.* 75.
6. What is the sum of 1, 4, 9, and 5 tens? *Ans.* 64.
7. What is the sum of 4, 8, 3, and 7 tens? *Ans.* 85.

8. If a top costs 6 cents, a knife 25 cents, a slate pencil 1 cent, and a slate 12 cents, what does the whole amount to? *Ans.* 44 *cts.*

9. John gives 30 cents for a bunch of quills, 18 cents for an inkstand, and 25 cents for a quire of paper, what did they all cost him? *Ans.* 73 *cts.*

10. Add together the numbers 894 and 637.

| | OPERATION. |
|---|---|
| Write the numbers thus - - - - - - - | 894 |
| | 637 |
| And draw a line beneath them - - - | —— |
| Sum of the column of units - - - - | 11 |
| Sum of the column of tens - - - - - | 12 |
| Sum of the column of hundreds - - - | 14 |
| Sum total - - - | 1531 |

In this example, the sum of the units is 11, which cannot be expressed by a single figure. But 11 units are equal to 1 ten and 1 unit; therefore, we set down 1 in the place of units, and 1 in the place of tens. The sum of the tens is 12. But 12 tens are equal to 1 hundred, and 2 tens; so that 1 is set down in the hundred's place, and 2 in the ten's place. The sum of the hundreds is 14. The 14 hundreds are equal to 1 thousand, and 4 hundreds; so that 4 is set down in the place of hundreds, and 1 in the place of thousands. The sum of these numbers, 1531, is the sum sought.

The example may be done in another way, thus: Having set down the numbers, as before, we say, 7 and 4 are 11: we set down 1 in the units place, and write the 1 ten under the 3 in the column of tens. We then say, 1 to 3 is four, and 9 are 13. We set down the three in the tens place, and write the 1 hundred under the 6 in the column of hundreds. We then add the 1, 6, and 8 together, for the hundreds, and find the entire sum 1531, as before.

| OPERATION. |
| ---: |
| 894 |
| 637 |
| 11 |
| 1531 |

When the sum in any one of the columns exceeds 10, or an exact number of tens, the excess must be written down, and a number equal to the number of tens, added to the next left hand column.

This is called *carrying to the next column.* The number to be carried may be written under the column or remembered and added in the mind. From these illustrations we deduce the following general

## RULE.

§ **14.** I. *Set down the numbers to be added, units under units, tens under tens, hundreds under hundreds, &c., and draw a line beneath them.*

II. *Begin at the foot of the unit's column, and add up the figures of that column. If the sum can be expressed by a single figure, write it beneath the line, in the unit's place. But if it cannot, see how many tens and how many units it contains. Write down the units in the unit's place, and carry as many to the bottom figure of the second column as there were tens in the sum. Add up that column: set down the sum and carry to the third column as before.*

III. *Add each column in the same way, and set down the entire sum of the last column.*

Q. How do you set down the numbers for addition? Where do you begin to add? If the sum of the first column can be expressed by a single figure, what do you do with it? When it cannot what do you write down? What do you then add to the next column? When you add the tens to the next column, what is it called? What do you set down when you come to the last column?

EXAMPLES.

1. What is the sum of the numbers 375, 6321 and 598.

In this example, the small figure placed under the 4, shows how many are to be carried from the first column to the second, and the small figure under the 9, how many are to be carried from the second column to the third.

OPERATION.

| |
|---:|
| 375 |
| 6321 |
| 598 |
| 7294 |
| 11 |

In like manner, in the examples below, the small figure under each column, shows how many are to be carried to the next column at the left. Beginners had better set down the numbers to be carried as in the examples.

| | (2.) | | (3.) | | (4.) |
|---|---:|---|---:|---|---:|
| | 96972 | | 9841672 | | 81325 |
| | 3741 | | 793139 | | 6784 |
| | 9299 | | 888923 | | 2130 |
| Sum | 110012 | Sum | 11523734 | Sum | 90239 |
| | 2221 | | 221111 | | 1110 |

PROOF OF ADDITION.

§ 15. Begin at the right hand figure of the upper line, and add all the columns downwards, carrying from one column to the other, as before. If the two results agree the work is supposed right.

SECOND PROOF.

Draw a line under the upper number. Add the lower numbers together, and then add their sum to the upper number. If the last sum is the same as the sum total, first found, the work may be regarded as right.

Q. What do the small figures under the columns denote? How do you prove addition by the first method? How do you prove addition by the second method?

EXAMPLES.

| | (1.) | | (2.) | | (3.) |
|---|---|---|---|---|---|
| | 34578 | | 22345 | | 23456 |
| | 3750 | | 67890 | | 78901 |
| | 87 | | 8752 | | 23456 |
| | 328 | | 340 | | 78901 |
| | 17 | | 350 | | 23456 |
| | 327 | | 78 | | 78901 |
| Sum | 39087 | Sum | 99755 | Sum | 307071 |
| | 4509 | | 77410 | | 283615 |
| Proof | 39087 | Proof | 99755 | Proof | 307071 |

| (4.) | (5.) | (6.) |
|---|---|---|
| 672981043 | 91278976 | 8416785413 |
| 67126459 | 7654301 | 6915123460 |
| 39412767 | 876120 | 31810213 |
| 7891234 | 723456 | 7367985 |
| 109126 | 31309 | 654321 |
| 84172 | 4871 | 37853 |
| 72120 | 978 | 2685 |
| 787676921 | 100570011 | 15371781930 |

7. Add 8635, 2194, 7421, 5063, 2196, and 1245 together. *Ans.* 26754.

8. Add 246034, 298765, 47321, 58653, 64218, 5376, 9821, and 340 together. *Ans.* 730528.

9. Add 27104, 32547, 10758, 6256, 704321, 730491, 2787316, and 2749104 together. *Ans.* 7047897.

10. Add 1, 37, 29504, 6790312, 18757421, and 265 together. *Ans.* 25577540.

11. Add 562163, 21964, 56321, 18536, 4340, 279, and 83 together. *Ans.* 663686.

12. What is the sum of the following numbers: viz., Seventy-five; one thousand and ninety-five; six thousand four hundred and thirty-five; two hundred and sixty-seven thousand; one thousand four hundred and fifty-five; twenty seven millions and eighteen; two hundred and seventy millions and twenty-seven thousand.

*Ans.* 297303078.

APPLICATIONS.

1. How many days are there in the twelve callendar months? January has 31, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, and December 31. *Ans.* 365 *days.*

2. A merchant on settling his accounts finds that he owes A 60 dollars, B 150 dollars, C 240 dollars, and to D 100 dollars. How much does he owe in all?

*Ans.* 550 *dollars.*

3. What is the total weight of seven casks of merchandise: viz. No. 1, weighing 960 pounds, No. 2, 725 pounds, No. 3, 830 pounds, No. 4, 798 pounds, No. 5, 698 pounds, No. 6, 569 pounds, No. 7, 987 pounds?

*Ans.* 5567 *pounds.*

4. A man borrowed a sum of money and paid in part 267 dollars, and afterwards paid the remainder 325 dollars: How much did he borrow? *Ans.* 592 *dollars.*

5. At the Custom House, on the first day of June, there were entered 1800 yards of linen; on the 10th, 2500 yards; on the 25th, 600 yards; on the day following, 7500 yards; and on the three last days of the month, 1325 yards each day: What was the whole amount entered during the month? *Ans.* 16375 *yards.*

6. A farmer has his live stock distributed in the following manner: in pasture No. 1, there are 5 horses, 14 cows, 8 oxen, and 6 colts; in pasture No. 2, 3 horses, 4 colts, 6 cows, 20 calves, and 12 head of young cattle; in pasture No. 4, 320 sheep, 16 calves, 2 colts, and 5 head of young cattle. How much live stock had he of each kind, and how many head had he altogether?

*Ans.* 8 *horses*, 20 *cows*, 8 *oxen*, 12 *colts*, 36 *calves*, 17 *head of young cattle, and* 320 *sheep.* Total live stock, 421 *head.*

7. What is the interval of time between a transaction which happened 125 years ago, and one that will happen 267 years hence? *Ans.* 392 *years.*

8. An army consists of 4000 foot soldiers, 4006 cavalry or horse, 3093 artillery-men, 1224 riflemen, 1400 pioneers

and 200 miners: What is the whole number of men in the army? *Ans.* 13923.

9. The mail route from Albany to New-York is 144 miles, from New-York to Philadelphia 90 miles, from Philadelphia to Baltimore 98 miles, and from Baltimore to Washington City 38 miles: What is the distance from Albany to Washington? *Ans.* 370 *miles.*

10. Suppose a man was born on the 1st of January, 1795: When will he be 85 years old? *Ans.* In 1880.

11. A man dying leaves his only daughter nine hundred and ninety-nine dollars, and to each of three sons two hundred dollars more than he left the daughter. What was each son's portion, and what the amount of the whole estate? *Ans.* { Each son's part 1199 *dollars.* Whole estate 4596 *dollars.* }

12. What was the whole number of inhabitants in the United States in 1830; there being in Maine 399,955; New Hampshire 269,328; Vermont 280,652; Massachusetts 610,408; Rhode Island 97,199; Connecticut 297,675; New York 1,918,608; New Jersey 320,823; Pennsylvania 1,348,233; Delaware 76,748; Maryland 447,040: Virginia 1,211,405; North Carolina 737,987; South Carolina 581,185; Georgia 516,823; Alabama 309,527; Mississippi 110,357; Louisiana 215,739; Tennessee 681,904; Kentucky 687,917; Ohio 937,903; Indiana 343,031; Illinois 157,445; Missouri 140,455; Michigan Territory 31,639; Arkansas 30,388; Florida 34,730; District of Columbia 39,834; Naval Service 5,602. *Ans.* 12,840 540.

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## SUBTRACTION OF SIMPLE NUMBERS.

§ **16**. John has 6 apples and Charles has 4. How many more apples has John than Charles? *Ans.* 2.

Two is called the *difference* between the number of apples which John has, and the number of apples which Charles has.

SUBTRACTION *is finding the difference between two numbers.*

The larger of the two numbers is called the *minuend*, the lesser is called the *subtrahend*, and their difference is called the *remainder*.

Q. What is Subtraction? What is the larger number called? What is the smaller number called? What is the difference called?

§ **17**. James has 8 pears and gives 5 to William: how many has he left? *Ans.* 3.

Q. Which number is the *minuend?* Which the *subtrahend ?* Which the *remainder ?*

§ **18**. The sign —, is called *minus*, a term signifying less. When placed between two numbers it denotes that the one on the right is to be taken from the one on the left.

Thus, 6—4=2, denotes that 4 is to be taken from 6. Here 6 is the minuend, 4 the subtrahend, and 2 the remainder.

When the numbers are small, their difference is apparent, and instead of saying, 6 minus 4 equals 2, we say, 4 from 6 leaves 2.

Q. What is the sign of subtraction? What is it called? What does the term signify? When placed between two numbers what does it denote? When the numbers are small how do you read them, as 6 — 4?

§ **19**. The following table should be committed to memory, and read, two from two naught remains; two from three, one remains, &c.

**SUBTRACTION TABLE.**

| | | | |
|---|---|---|---|
| 2—2=0 | 3—3=0 | 4—4=0 | 5—5=0 |
| 3—2=1 | 4—3=1 | 5—4=1 | 6—5=1 |
| 4—2=2 | 5—3=2 | 6—4=2 | 7—5=2 |
| 5—2=3 | 6—3=3 | 7—4=3 | 8—5=3 |
| 6—2=4 | 7—3=4 | 8—4=4 | 9—5=4 |
| 7—2=5 | 8—3=5 | 9—4=5 | 10—5=5 |
| 8—2=6 | 9—3=6 | 10—4=6 | 11—5=6 |
| 9—2=7 | 10—3=7 | 11—4=7 | 12—5=7 |
| 10—2=8 | 11—3=8 | 12—4=8 | 13—5=8 |
| 11—2=9 | 12—3=9 | 13—4=9 | 14—5=9 |
| 12—2=10 | 13—3=10 | 14—4=10 | 15—5=10 |

| | | | |
|---|---|---|---|
| 6−6=0 | 7−7=0 | 8−8=0 | 9−9=0 |
| 7−6−1 | 8−7=1 | 9−8=1 | 10−9=1 |
| 8−6=2 | 9−7=2 | 10−8=2 | 11−9=2 |
| 9−6=3 | 10−7=3 | 11−8=3 | 12−9=3 |
| 10−6=4 | 11−7=4 | 12−8=4 | 13−9=4 |
| 11−6=5 | 12−7=5 | 13−8=5 | 14−9=5 |
| 12−6=6 | 13−7=6 | 14−8=6 | 15−9=6 |
| 13−6=7 | 14−7=7 | 15−8=7 | 16−9=7 |
| 14−6=8 | 15−7=8 | 16−8=8 | 17−9=8 |
| 15−6=9 | 16−7=9 | 17−8=9 | 18−9=9 |
| 16−6=10 | 17−7=10 | 18−8=10 | 19−9=10 |

| | |
|---|---|
| 12−2=10 | 17−7= how many? |
| 12−3= how many? | 16−8= how many? |
| 15−4= how many? | 19−9= how many? |
| 11−6= how many? | 20−4= how many? |
| 18−9= how many? | 13−7= how many? |
| 25−8= how many? | 14−2= how many? |

### EXAMPLES.

1. From the number 869 subtract 327.

OPERATION.

| | |
|---|---|
| 869 | Minuend. |
| 327 | Subtrahend. |
| 542 | Remainder. |

We begin at the right hand figure of the lower line, and say, 7 from 9 leaves 2: set down the 2 under the 7. Proceeding to the next column, we say, 2 from 6 leaves 4: set down the 4, and then say, 3 from 8 leaves 5. Thus, 542 is the remainder, or true difference between the numbers.

2. From 654 subtract 472.

OPERATION.

| hundreds. | tens | units | | hundreds. | tens | units |
|---|---|---|---|---|---|---|
| 6 | 5 | 4 | | 5 | 15 | 4 |
| 4 | 7 | 2 | | 4 | 7 | 2 |
| 1 | 8 | 2 | | 1 | 8 | 2 |

Beginning at the lower figure on the right, we say, 2 from 4 leaves 2: set down the 2. At the next step we meet a difficulty; for, we cannot subtract 7 from 5. We avoid this difficulty, thus. Ten units in the second place are equal to one unit of the third place § 8. Therefore, if we add 10 to the 5 and diminish 6 by 1 the value of the upper line will not be changed. The numbers are so written at the right.

Now, instead of saying 7 from 5, we say, 7 from 15 leaves 8: set down the 8, and then say, 4 from 5 leaves 1. The remainder is, therefore, 182.

Now, if instead of diminishing the 6 by 1, we had increased the 4 under it by 1, and subtracted 5 from 6, the remainder would have been the same. Therefore,

*When a figure of the subtrahend is greater than the one directly over it, suppose* 10 *to be added to* the upper *figure. Let the lower figure be then taken from the number thus arising, and add* 1 *to the next figure of the subtrahend before it is subtracted from the figure directly above it. This is called borrowing* 10.

Q. When a figure of the subtrahend is greater than the one of the minuend directly above it, what do you do? What is this called?

3. From 6354 subtract 4627.

In this example, we say, 7 from 14 leaves 7: 1 carried to 2 is 3, 3 from 5 leaves 2: 6 from 13 leaves 7: 1 carried to 4 is 5, 5 from 6 leaves 1. The remainder, therefore, is 1727.

OPERATION.

| thous. | huns. | tens. | units. | thous. | huns. | tens. | units. |
|---|---|---|---|---|---|---|---|
| 6 | 3 | 5 | 4 | 5 | 13 | 4 | 14 |
| 4 | 6 | 2 | 7 | 4 | 6 | 2 | 7 |
| 1 | 7 | 2 | 7 | 1 | 7 | 2 | 7 |

4. From 60204 subtract 32861.

In this example, we say, 1 from 4 leaves 3: 6 from 10 leaves 4: 1 carried to 8 is 9, 9 from 12 leaves 3: 1 carried to 2 is 3, 3 from 10 leaves 7: 1 carried to 3 is 4, 4 from 6 leaves 2.

OPERATION.

```
 60204
 32861
 27343
```

From these examples, we may deduce the following general

### RULE.

§ **20.** I. *Set down the less number under the greater, so that units shall fall under units, tens under tens, hundreds under hundreds, &c., and draw a line beneath them.*

II. *Then, beginning at the right hand, subtract each figure from the one directly over it, and set down the remainder.*

II. *But if the upper figure be the least, suppose it to be increased by* 10*: then make the subtraction, set down the remainder, and carry* 1 *to the next figure of the subtrahend.*

### PROOF.

Add the remainder to the subtrahend. If their sum is equal to the minuend the work may be regarded as right.

Q. How do you set down the numbers for subtraction? Where do you begin to subtract? How do you subtract? How do you prove subtraction?

### EXAMPLES.

| | (1.) | (2.) | (3.) |
|---|---|---|---|
| Minuends | 8592678 | 67942139 | 219067803 |
| Subtrahends | 1078953 | 9756783 | 104202196 |
| Remainders | 7513725 | 58185356 | 114865607 |
| Proofs. | 8592678 | 67942139 | 219067803 |

| | (4.) | (5.) | (6.) | (7.) | (8.) |
|---|---|---|---|---|---|
| | 10000 | 30000 | 67987 | 100000 | 87000 |
| | 4 | 9999 | 40000 | 1 | 1009 |
| Remainders | 9996 | 20001 | 27987 | 99999 | 85991 |

9. From 2637804 take 2376982. *Ans.* 260822.

10. From 3762162 take 826541. *Ans.* 2935621.

11. From 78213609 take 27821890. *Ans.* 50391716.

12. From thirty thousand and ninety-seven, take one thousand six hundred and fifty-four. *Ans.* 28443.

13. From one hundred million two hundred and forty seven thousand, take one million four hundred and nine. *Ans.* 99246591.

14. Subtract one from one million. *Ans.* 999999.

### APPLICATIONS.

1. Suppose John were born in eighteen hundred and fifteen, and James in eighteen hundred and twenty-five: what is the difference of their ages? *Ans.* 10 *years.*

2. A man was born in 1785: what was his age in 1830? *Ans.* 45 *years.*

3. Suppose I lend a man 1565 dollars, and he dies, owing me 450 dollars: how much had he paid me? *Ans.* 1115 *dollars.*

4. In five bags are different sums of money to the amount in all of 1000 dollars. In the first there are 100 dollars; in the second, 314 dollars; in the third, 143 dollars; and in the fourth, 209 dollars: how many dollars does the fifth contain? *Ans.* 234 *dollars.*

5. America was discovered by Christopher Columbus in the year 1492. What number of years has since elapsed?

6. George Washington was born in the year 1732, and died in 1799: how old was he at the time of his death? *Ans.* 67 *years.*

7. The declaration of independence was published July 4th, 1776: how many years to July 4th, 1838? *Ans.* 62 *years.*

8. By the census of 1830, it appeared, that the white population of the United States was 10,526,248, and the number of blacks 2,328,642: how much did the white population exceed the black? *Ans.* 8,197,606.

9. In 1830 there were in the State of New York 1,918,608 inhabitants, and in the State of Pennsylvania 1,348,233 inhabitants: how many more inhabitants were there in New York than in Pennsylvania? *Ans.* 570,375.

10. The revolutionary war began in 1775; the late war in 1812: what time elapsed between their commencements? *Ans.* 37 *years.*

11. In 1830 there were in New York, (which is the largest city in the United States,) 207,021 inhabitants, and in Philadelphia, (the next largest city,) 161,412: how many more inhabitants were there in New York, than in Philadelphia? *Ans.* 45,609.

12. A man dies worth 1200 dollars; he leaves 504 to his daughter, and the remainder to his son: what was the son's portion? *Ans.* 696 *dollars.*

13. Suppose a gentleman has an income of 3090 dollars a year, and pays for taxes 150 dollars, and expends besides 253 dollars: how much does he lay up? *Ans.* 2687 *dollars.*

14. A merchant bought 500 barrels of flour for 3500 dollars; he sold 250 barrels for 2000 dollars: how many

barrels remained on hand, and how much must he sell them for, that he may lose nothing?

*Ans.* 250 *barrels* remained, and he must sell for 1500 *dollars.*

APPLICATIONS IN ADDITION AND SUBTRACTION.

1. A merchant buys 19576 yards of cloth of one person, 27580 yards of another, and 375 yards of a third: he sells 1050 yards to one customer, 6974 yards to another, and 10462 yards to a third: how many yards has he remaining? *Ans.* 29045.

2. A person borrowed of his neighbour at one time 355 dollars, at another time 637 dollars, and 403 dollars at another time: he then paid him 977 dollars. How much did he owe him? *Ans.* 418.

3. I have a fortune of 2543 dollars to divide among my four sons, James, John, Henry, and Charles. I give James 504 dollars, John 600 dollars, and Henry 725: how much remains for Charles? *Ans.* 714 *dollars.*

4. I have a yearly income of ten thousand dollars. I pay 275 for rent, 220 dollars for fuel, 35 dollars to the doctor, and 3675 dollars for all my other expenses: how much have I left at the end of the year? *Ans.* 5795.

5. A man pays 300 dollars for 100 sheep, 95 dollars for a pair of oxen, 60 dollars for a horse, and 125 dollars for a chaise. He gives in return 100 bushels of wheat worth 125 dollars, a cow worth 25 dollars, a colt worth 40 dollars, and pays the rest in cash: what amount of money does he pay? *Ans.* 390 *dollars.*

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## MULTIPLICATION OF SIMPLE NUMBERS.

§ **21**. If Charles gives 2 cents apiece for two oranges: how much do they cost him? *Ans.* 4 *cents.*

If Charles gives 2 cents apiece for 3 oranges: how much do they cost him? *Ans.* 6 *cents.*

If he gives 2 cents apiece for 4 oranges: how much do they cost him? *Ans.* 8 *cents.*

If he gives 2 cents apiece for 5 oranges: how much do they cost him? *Ans.* 10 *cents.*

The cost in each case, may be obtained by adding the price of the separate oranges; thus,

$2+2=4$ cents, the cost of 2 oranges,
$2+2+2=6$ cents, the cost of 3 oranges,
$2+2+2+2=8$ cents, the cost of 4 oranges,
$2+2+2+2+2=10$ cents, the cost of 5 oranges.

In the first case 2 is repeated *two times*, in the second case it is repeated *three times*, in the third, *four times*, and in the fourth it is repeated *five times*; and in a similar manner any number may be repeated as often as we please by adding it continually to itself.

MULTIPLICATION *is a short method of repeating one number as many times as there are units in another.*

The number to be repeated is called the *multiplicand.*

The number denoting how many times the multiplicand is to be repeated, is called the *multiplier.*

The number arising from repeating the multiplicand as many times as there are units in the multiplier, is called the *product.*

The multiplicand and multiplier are called *factors*, or *producers* of the *product.*

The sign ×, placed between two numbers, denotes that they are to be multiplied together. It is called, the *sign of multiplication.*

Q. What is multiplication? What is the number called which is to be repeated? What does the multiplier denote? What is the product? In the case of the two oranges, which is the multiplicand? Which is the multiplier? Which is the product? In the case of three oranges, which is the multiplicand, which the multiplier, and which the product? What are the multiplicand and multiplier called? How do you denote that two numbers are to be multiplied together? What is the sign called?

## MULTIPLICATION TABLE.

| | | |
|---|---|---|
| 1 times 0 is 0 | 4 times 0 are 0 | 7 times 0 are 0 |
| 1 times 1 is 1 | 4 times 1 are 4 | 7 times 1 are 7 |
| 1 times 2 is 2 | 4 times 2 are 8 | 7 times 2 are 14 |
| 1 times 3 is 3 | 4 times 3 are 12 | 7 times 3 are 21 |
| 1 times 4 is 4 | 4 times 4 are 16 | 7 times 4 are 28 |
| 1 times 5 is 5 | 4 times 5 are 20 | 7 times 5 are 35 |
| 1 times 6 is 6 | 4 times 6 are 24 | 7 times 6 are 42 |
| 1 times 7 is 7 | 4 times 7 are 28 | 7 times 7 are 49 |
| 1 times 8 is 8 | 4 times 8 are 32 | 7 times 8 are 56 |
| 1 times 9 is 9 | 4 times 9 are 36 | 7 times 9 are 63 |
| 1 times 10 is 10 | 4 times 10 are 40 | 7 times 10 are 70 |
| 1 times 11 is 11 | 4 times 11 are 44 | 7 times 11 are 77 |
| 1 times 12 is 12 | 4 times 12 are 48 | 7 times 12 are 84 |
| 2 times 0 are 0 | 5 times 0 are 0 | 8 times 0 are 0 |
| 2 times 1 are 2 | 5 times 1 are 5 | 8 times 1 are 8 |
| 2 times 2 are 4 | 5 times 2 are 10 | 8 times 2 are 16 |
| 2 times 3 are 6 | 5 times 3 are 15 | 8 times 3 are 24 |
| 2 times 4 are 8 | 5 times 4 are 20 | 8 times 4 are 32 |
| 2 times 5 are 10 | 5 times 5 are 25 | 8 times 5 are 40 |
| 2 times 6 are 12 | 5 times 6 are 30 | 8 times 6 are 48 |
| 2 times 7 are 14 | 5 times 7 are 35 | 8 times 7 are 56 |
| 2 times 8 are 16 | 5 times 8 are 40 | 8 times 8 are 64 |
| 2 times 9 are 18 | 5 times 9 are 45 | 8 times 9 are 72 |
| 2 times 10 are 20 | 5 times 10 are 50 | 8 times 10 are 80 |
| 2 times 11 are 22 | 5 times 11 are 55 | 8 times 11 are 88 |
| 2 times 12 are 24 | 5 times 12 are 60 | 8 times 12 are 96 |
| 3 times 0 are 0 | 6 times 0 are 0 | 9 times 0 are 0 |
| 3 times 1 are 3 | 6 times 1 are 6 | 9 times 1 are 9 |
| 3 times 2 are 6 | 6 times 2 are 12 | 9 times 2 are 18 |
| 3 times 3 are 9 | 6 times 3 are 18 | 9 times 3 are 27 |
| 3 times 4 are 12 | 6 times 4 are 24 | 9 times 4 are 36 |
| 3 times 5 are 15 | 6 times 5 are 30 | 9 times 5 are 45 |
| 3 times 6 are 18 | 6 times 6 are 36 | 9 times 6 are 54 |
| 3 times 7 are 21 | 6 times 7 are 42 | 9 times 7 are 63 |
| 3 times 8 are 24 | 6 times 8 are 48 | 9 times 8 are 72 |
| 3 times 9 are 27 | 6 times 9 are 54 | 9 times 9 are 81 |
| 3 times 10 are 30 | 6 times 10 are 60 | 9 times 10 are 90 |
| 3 times 11 are 33 | 6 times 11 are 66 | 9 times 11 are 99 |
| 3 times 12 are 36 | 6 times 12 are 72 | 9 times 12 are 108 |

| | | |
|---|---|---|
| 10 times 0 are 0 | 11 times 0 are 0 | 12 times 0 are 0 |
| 10 times 1 are 10 | 11 times 1 are 11 | 12 times 1 are 12 |
| 10 times 2 are 20 | 11 times 2 are 22 | 12 times 2 are 24 |
| 10 times 3 are 30 | 11 times 3 are 33 | 12 times 3 are 36 |
| 10 times 4 are 40 | 11 times 4 are 44 | 12 times 4 are 48 |
| 10 times 5 are 50 | 11 times 5 are 55 | 12 times 5 are 60 |
| 10 times 6 are 60 | 11 times 6 are 66 | 12 times 6 are 72 |
| 10 times 7 are 70 | 11 times 7 are 77 | 12 times 7 are 84 |
| 10 times 8 are 80 | 11 times 8 are 88 | 12 times 8 are 96 |
| 10 times 9 are 90 | 11 times 9 are 99 | 12 times 9 are 108 |
| 10 times 10 are 100 | 11 times 10 are 110 | 12 times 10 are 120 |
| 10 times 11 are 110 | 11 times 11 are 121 | 12 times 11 are 132 |
| 10 times 12 are 120 | 11 times 12 are 132 | 12 times 12 are 144 |

### EXAMPLES.

1. Let it be required to multiply 4 by 2. Here 4 is the multiplicand and 2 is the multiplier, and it is required to find the product, which is the number arising from repeating 4 two times.

The product of 4 by 2 is found by multiplication, or by adding two 4's together.

OPERATION.

Multiplicand. Multiplier.

$$4 \times 2 = \begin{array}{r} 4 \\ 4 \\ \hline 8 \end{array} \text{ Product.}$$

2. Let it be required to multiply 4 by 3, and also to multiply 5 by 3.

OPERATION.

Multiplicand. Multiplier.

$$4 \times 3 = \left\{ \begin{array}{r} 4 \\ 4 \\ 4 \\ \hline 12 \end{array} \right. \text{ Product.}$$

OPERATION.

Multiplicand. Multiplier.

$$5 \times 3 = \left\{ \begin{array}{r} 5 \\ 5 \\ 5 \\ \hline 15 \end{array} \right. \text{ Product.}$$

From these examples we see, that the product of 4 multiplied by 3 is 12, the number which arises from adding three 4's together; and that the product of 5 by 3 is equal to 15, the number which arises from adding three 5's together.

We see from the above examples, that any product may be found by setting down the multiplicand as many times as there are units in the multiplier, and adding all the numbers together.

MULTIPLICATION *is therefore a short method of addition.*

Q. How may any product be found? What may multiplication be considered?

§ **22**. In the example in which 5 was multiplied by 3, the product was 15. Now, had we multiplied 3 by 5, the product would still have been 15. For, place as many ones in each horizontal row as there are units in the multiplicand, and make as many rows as there are units in the multiplier: the product will then be equal to the whole number of ones: viz., 15. But if we consider the number of ones (3) in a vertical row to be the multiplicand, and the number of vertical rows (5) the multiplier, the product will still be the whole number of ones: viz., 15. Hence,

OPERATION.

$$3\left\{\begin{array}{ccccc} \multicolumn{5}{c}{\overbrace{\hphantom{1\quad1\quad1\quad1\quad1}}^{5}} \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right.$$

*Either of the factors may be used as the multiplier without altering the product.* For example,

$3\times7=7\times3=21$: also, $6\times3=3\times6=18$.
$9\times5=5\times9=45$: also, $8\times6=6\times8=48$.
and, $8\times7=7\times8=56$: also, $5\times7=7\times5=35$.

Q. Is the product of two numbers altered by changing the multiplicand into the multiplier, and the multiplier into the multiplicand? Is 7 multiplied by 8 the same as 8 multiplied by 7?

3. Multiply 236 by 4.

First set down the 236, then place the 4 under the unit's place 6, and draw a line beneath it. Then multiply the 6 by 4: the product is 24 units; set them down. Next multiply the 3 tens by 4: the product is 12 tens; set down the 2 under the tens of the 24, leaving the 1 to the left, which is the place of

OPERATION.

| | |
|---|---|
| 236 | |
| 4 | |
| 24 | units. |
| 12 | tens. |
| 8 | hundreds. |
| 944 | |

hundreds. Next multiply the 2 by 4: the product is 8, which being hundreds, is set down under the 1. The The sum of these numbers, 944, is the entire product.

The product can also be found, thus: say 4 times 6 are 24: set down the 4, and then say, 4 times 3 are 12 and 2 to carry are 14: set down the 4, and then say, 4 times 2 are 8 and 1 to carry are 9. Set down the 9, and the product is 944 as before.

OPERATION.

```
236
  4
---
944
```

4. Multiply 627 by 84.

Multiply by the 4 units, as in the last example. Then multiply by the 8 tens. The first product 56, is 56 tens; the 6, therefore, must be set down under the 0, which is the place of tens, and the 5 carried to the product of the 2 by 8. Then multiply the 6 by 8, carrying the 2 from the last product, and set down the result 50. The sum of the numbers 52668, is the required product.

OPERATION.

```
  627
   84
-----
 2508
5016
-----
52668
```

5. Multiply 506 by 302.

In this example, we say, 2 times 6 are 12: then set down the 2, and say, 2 times 0 are 0 and 1 to carry make 1. Set down the 1, and say, 2 times 5 are 10: set down the 10. Then beginning with the 0, we say, 0 times 6 is 0: set down the 0. Then say, 0 times 0 is 0; set down the 0, and then say, 0 times 5 is 0. Then multiply by the 3 hundreds and set down the first figure 8 in the place of hundreds, and place the other figures to the left.

OPERATION.

```
   506
   302
------
  1012
 000
1518
------
152812
```

*When an* 0 *appears in the multiplier, we need not multiply by it, since each of the products is* 0; *but when we multiply by the next figure to the left, we must observe to set the first figure of the product directly under its multiplier.*

Thus, we have placed 8 directly under the multiplier 3.

Q. When an 0 is found in the multiplier need you multiply by it? When you multiply by the next figure to the left, where do you place the first figure of the product?

CASE I.

§ **23**. When the multiplier does not exceed 12.

RULE.

I. *Set down the multiplicand and under it set the multiplier, so that units shall fall under units, and draw a line beneath.*

II. *Multiply every figure of the multiplicand by the multiplier, setting down and carrying as in addition.*

EXAMPLES.

| (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|
| 867901 | 278904 | 678741 | 3021945 |
| 1 | 2 | 3 | 4 |
| 867901 | 557808 | 2036223 | 12087780 |

| (5.) | (6.) | (7.) | (8.) |
|---|---|---|---|
| 28432 | 82798 | 6789 | 49604 |
| 8 | 9 | 11 | 12 |
| 227456 | 745182 | 74679 | 595248 |

Q. When the multiplier does not exceed 12, how do you set it down? How do you multiply by it?

CASE II.

§ **24**. When the multiplier exceeds 12.

RULE.

I. *Set down the multiplier under the multiplicand, so that units shall fall under units, tens under tens, &c., and draw a line beneath.*

II. *Begin with the right hand figure, and multiply all the figures of the multiplicand by each figure of the multiplier, and when any of the products exceed 9, set down and carry to the next product as in addition; observing to write the first figure of each product directly under its multiplier.*

III. *Add up the several products and their sum will be the product sought.*

NOTE. There are three numbers in every multiplication. First, the multiplicand: second, the multiplier: and third, the product.

### PROOF OF MULTIPLICATION.

Write the multiplicand in the place of the multiplier, and find the product as before: if the two products are the same, the work is supposed right.

Q. When the multiplier exceeds 12, how do you set it down? How do you multiply by it? How do you add up? How many numbers are there in every multiplication? Name them? How do you prove multiplication.

### EXAMPLES.

1. Multiply 365 by 84: also, 37864 by 209.

| | (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|---|
| Multiplicand | 365 | 37864 | 34293 | 47042 |
| Multiplier | 84 | 209 | 74 | 91 |
| | 1460 | 340776 | | |
| | 2920 | 75728 | | |
| Product. | 30660 | 7913576 | 2537682 | 4280822 |

| (5.) | (6.) | (7.) | (8.) |
|---|---|---|---|
| 46834 | 679084 | 1098731 | 8971432 |
| 406 | 126 | 1987 | 10471 |
| 19014604 | 85564584 | 2183178497 | 93939864472 |

9. Multiply 12345678 by 32. *Ans.* 395061696.
10. Multiply 9378964 by 42. *Ans.* 393916488.
11. Multiply 1345894 by 49. *Ans.* 65948806.
12. Multiply 576784 by 64. *Ans.* 36914176.
13. Multiply 596875 by 144. *Ans.* 85950000.
14. Multiply 46123101 by 72. *Ans.* 3320863272.
15. Multiply 61835720 by 132. *Ans.* 8162315040.
16. Multiply 718328 by 96. *Ans.* 68959488.
17. Multiply 7128368 by 1440. *Ans.* 10264849920.
18. Multiply 6795634 by 918546. *Ans.* 6242102428164.
19. Multiply 86972 by 1208. *Ans.* 105062176.
20. Multiply 1055054 by 570. *Ans.* 601380780.
21. Multiply 538362 by 9258. *Ans.* 4984155396.
22. Multiply 50406 by 8050. *Ans.* 405768300.
23. Multiply 523972 by 15276. *Ans.* 8004196272.
24. Multiply 760184 by 16150. *Ans.* 12276971600.
25. Multiply 1055070 by 31456. *Ans.* 33188281920

CASE III.

§ **25**. When the multiplier is 1 and any number of ciphers after it, as 10, 100, 1000, &c.

Placing a cipher on the right of a number changes the units place into tens, the tens into hundreds, the hundreds into thousands, &c., and *therefore increases the number ten times.*

Thus, 5 is increased ten times by making it 50. So the addition of two ciphers increases a number *one hundred times;* the addition of three ciphers, *a thousand times, &c.*

Thus, 6 is increased a hundred times by making it 600, and 5 is increased a thousand times, by making it 5000.

Hence, we have the following

RULE.

*Place on the right of the multiplicand as many ciphers as there are in the multiplier, and the number so formed will be the required product.*

Q. If you place one cipher on the right of a number, what effect has it on its value? If you place two, what effect has it? If you place three? And for any number of ciphers, how much will each increase it? How do you multiply by 10, 100, 1000, &c.?

EXAMPLES.

1. Multiply 254 by 10. *Ans.* 2540.
2. Multiply 648 by 100 *Ans.* 64800.
3. Multiply 7987 by 1000. *Ans.* 7987000.
4. Multiply 9840 by 10000 *Ans.* 98400000.
5. Multiply 3750 by 100. *Ans.* 375000.

CASE IV.

§ **26**. When there are ciphers on the right hand of one or both of the factors.

RULE.

*Neglect the ciphers and multiply the significant figures: then place as many ciphers to the right hand of the product, as there are in both of the factors.*

Q. When there are ciphers on the right hand of both the factors, how do you multiply?

EXAMPLES.

| (1.) | (2.) | (3.) |
|---:|---:|---:|
| 76400 | 7532000 | 416000 |
| 24 | 580 | 357000 |
| 1833600 | 4368560000 | 148512000000 |

4. $4871000 \times 270000$. *Ans*. 1315170000000.
5. $296200 \times 875000$. *Ans*. 259175000000.
6. $3456789 \times 567090$. *Ans*. 1960310474010.
7. $21200 \times 70$. *Ans*. 1484000.
8. $359260 \times 304000$. *Ans*. 109215040000.
9. $7496430 \times 695000$. *Ans*. 5210018850000.

CASE V.

§ **27**. When the multiplier is a composite number.

A composite number is one that may be produced by the multiplication of two or more numbers, which are called the *components* or *factors*. Thus, $2 \times 3 = 6$. Here 6 is the composite number, and 2 and 3 are the factors, or components. The number $16 = 8 \times 2$: here 16 is a composite number, and 8 and 2 are the factors; and since $4 \times 4 = 16$, we may also regard 4 and 4 as factors or components of 16.

Q. What is a composite number? Is 6 a composite number? What are its components or factors? What are the factors of the composite number 16? What are the factors of the composite number 12?

EXAMPLES.

1 Let it be required to multiply 8 by the composite number 6, in which the factors are 2 and 3.

8

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | $2 \times 8 = 16$ |
| | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |

$$\begin{array}{r} 16 \\ 3 \\ \hline 48 \end{array} \qquad \begin{array}{r} 8 \\ 3 \\ \hline 24 \\ 2 \\ \hline 48 \end{array}$$

If we write 6 horizontal lines with 8 units in each, it is evident that the product of $8 \times 6 = 48$, the number of units in all the lines.

But let us first connect the lines in sets of 2 each, as on the right; there will then be in each set $8 \times 2 = 16$; or 16 units in each set. But there are 3 sets; hence, $16 \times 3 = 48$, the number of units in all the sets.

If we divide the lines into sets of 3 each, as on the left, the number of units in each set will be equal to $8 \times 3 = 24$, and there being 2 sets, $24 \times 2 = 48$, the whole number of units. As the same may be shown for all numbers we have the following

### RULE.

*When the multiplier is a composite number, multiply by by each of the factors in succession, and the last product will be the entire product sought.*

### EXAMPLES.

1. Multiply 327 by 12.

The factors of 12, are 2 and 6, or they are 3 and 4, or they are 3, 2 and 2: for, $2 \times 6 = 12$, $3 \times 4 = 12$, and $3 \times 2 \times 2 = 12$.

| | | | |
|---|---|---|---|
| | 327 | 327 | 327 |
| | 6 | 3 | 3 |
| | 1962 | 981 | 981 |
| | 2 | 4 | 2 |
| Product. | 3924 | 3924 | 1962 |
| | | | 2 |
| | | Product. | 3924 |

2. Multiply 5709 by 48; the factors being 8 and 6, or 16 and 3. *Ans.* 274032.
3. Multiply 342516 by 56. *Ans.* 19180896.
4. Multiply 209402 by 72. *Ans.* 15076944.
5. Multiply 937387 by 54. *Ans.* 50618898.
6. Multiply 91738 by 81. *Ans.* 7430778.
7. Multiply 3842 by 144. *Ans.* 553248.

### APPLICATIONS.

1. There are ten bags of coffee, each containing 48 pounds: how much coffee is there in all the bags?
*Ans.* 480 *pounds.*

2. There are 20 pieces of cloth each containing 37 yards; and 49 other pieces, each containing 75 yards: how many yards of cloth are there in all the pieces?

*Ans.* 4415 *yards.*

3. There are 24 hours in a day, and 7 days in a week: how many hours in a week? *Ans.* 168 *hours.*

4. A merchant buys a piece of cloth containing 97 yards, at 3 dollars a yard: what does the piece cost him?

*Ans.* 291 *dollars.*

5. A farmer bought a farm containing 10 fields; three of the fields contained 9 acres each; three other of the fields 12 acres each; and the remaining 4 fields, each 15 acres: how many acres were there in the farm, and how much did the whole cost at 18 dollars an acre?

*Ans.* { The farm contained 123 *acres.* It cost 2214 *dollars.* }

6. A merchant bought 49 hogsheads of molasses, each containing 63 gallons: how many gallons of molasses were there in the parcel? *Ans.* 3087 *gallons.*

7. Suppose a man were to travel 32 miles a day: how far would he travel in 365 days? *Ans.* 11680 *miles.*

8. In a certain city, there are 3751 houses. If each house on an average contains 5 persons, how many inhabitants are there in the town? *Ans.* 18755 *inhabitants.*

9. When a person sells goods he generally gives with them a bill, showing the amount charged for them, and acknowledging the receipt of the money paid; such bills are usually called *Bills of Parcels.*

BILLS OF PARCELS.

*New-York, Oct.* 1, 1838.

*James Johnson* *Bought of W. Smith.*

4 Chests of tea, of 45 pounds each, at 1 doll. a pound.
3 Firkins of butter at 17 dolls. per firkin - - -
4 Boxes of raisins at 3 dolls. per box - - - -
36 Bags of coffee at 16 dolls. each - - - - - -
14 Hogsheads of molasses at 28 dolls. each. - -

Amount 1211 *dollars.*

Received the amount in full,

*W. Smith.*

*Hartford, Nov.* 1, 1837.

*James Hughes* *Bought of W. Jones.*

27 Bags of coffee at 14 dolls. per bag - - - - -
18 Chests of tea at 25 dolls. per chest - - - - -
75 Barrels of shad at 9 dolls. per barrel - - - - -
87 Barrels of mackerel at 8 dolls. per barrel - -
67 Cheeses at 2 dolls. each - - - - - - - - -
59 Hogsheads of molasses at 29 dolls per hogshead.

Amount 4044 *dollars.*

Received the amount in full, for W. Jones,
per *James Cross.*

---

## DIVISION OF SIMPLE NUMBERS.

§ **28**. Charles has 12 apples and wishes to divide them equally between his four brothers.

He gives one to each, which takes 4. Subtracting 4 from 12, 8 remains. He then gives another to each, which takes 4 more. Subtracting this 4 from 8 leaves 4. He then gives one more to each, which takes all his apples, and leaves nothing. He has then divided them equally, and found that 12 *contains* 4, three times, for he has three times *subtracted* 4 from 12.

OPERATION.
```
12
 4
--
 8 1st remain.
 4
--
 4 2d remain.
 4
--
 0 3d remain.
--
```

Suppose he had 28 apples and wished to divide them equally among 8 boys.

Giving each one, would take 8 and leave 20. Giving each one, a second time, would take 8 and leave 12. Giving each one, a third time, would take 8 and leave 4. Hence, 8 is contained three times in 28, and there are 4 over.

OPERATION.
```
28
 8
--
20 1st remain.
 8
--
12 2d remain.
 8
--
 4 3d remain.
--
```

By continued subtraction we can always find how many times one number is contained in another, and also, what is left when it is not contained an exact number of times.

We can arrive at the same result by a shorter method, called *Division.*

DIVISION *teaches the manner of finding how many times a less number is contained in a greater. It is a short method of subtraction.*

The less number is called the *divisor.*

The greater number is called the *dividend.*

The number expressing how many times the dividend contains the divisor, is called the *quotient.*

If there is a number left, it is called the *remainder,* which is always less then the *divisor.*

There are three signs used to denote division. They are the following.

$18 \div 4$ expresses that 18 is to be divided by 4.
$\frac{18}{4}$ expresses that 18 is to be divided by 4.
4)18 expresses that 18 is to be divided by 4.

When the last sign is used, a curved line is also drawn on the right of the dividend to separate it from the quotient, which is generally set down on the right.

Q. When Charles divides 12 apples equally, among his four brothers, how many does he give to each? How many times does 12 contain 4? In dividing 28 apples equally among 8 boys, how many does each receive? How many remain? Which number is the dividend? Which the divisor? Which the quotient? Which the remainder? What does division teach? What is the less number called? What is the greater called? What is the answer called? What is the number called which is left? Is this number greater or less than the divisor? How many signs are there in division? Make them.

§ 29. Let the following table be committed to memory. It is read 2 in 2, 1 time; 2 in 4, 2 times, &c.

DIVISION TABLE.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 in | 2 | 1 | time | 3 in | 3 | 1 | time | 4 in | 4 | 1 | time |
| 2 in | 4 | 2 | times | 3 in | 6 | 2 | times | 4 in | 8 | 2 | times |
| 2 in | 6 | 3 | times | 3 in | 9 | 3 | times | 4 in | 12 | 3 | times |
| 2 in | 8 | 4 | times | 3 in | 12 | 4 | times | 4 in | 16 | 4 | times |
| 2 in | 10 | 5 | times | 3 in | 15 | 5 | times | 4 in | 20 | 5 | times |
| 2 in | 12 | 6 | times | 3 in | 18 | 6 | times | 4 in | 24 | 6 | times |
| 2 in | 14 | 7 | times | 3 in | 21 | 7 | times | 4 in | 28 | 7 | times |
| 2 in | 16 | 8 | times | 3 in | 24 | 8 | times | 4 in | 32 | 8 | times |
| 2 in | 18 | 9 | times | 3 in | 27 | 9 | times | 4 in | 36 | 9 | times |

| | | |
|---|---|---|
| 5 in 5 1 time | 8 in 8 1 time | 11 in 11 1 time |
| 5 in 10 2 times | 8 in 16 2 times | 11 in 22 2 times |
| 5 in 15 3 times | 8 in 24 3 times | 11 in 33 3 times |
| 5 in 20 4 times | 8 in 32 4 times | 11 in 44 4 times |
| 5 in 25 5 times | 8 in 40 5 times | 11 in 55 5 times |
| 5 in 30 6 times | 8 in 48 6 times | 11 in 66 6 times |
| 5 in 35 7 times | 8 in 56 7 times | 11 in 77 7 times |
| 5 in 40 8 times | 8 in 64 8 times | 11 in 88 8 times |
| 5 in 45 9 times | 8 in 72 9 times | 11 in 99 9 times |
| 6 in 6 1 time | 9 in 9 1 time | 12 in 12 1 time |
| 6 in 12 2 times | 9 in 18 2 times | 12 in 24 2 times |
| 6 in 18 3 times | 9 in 27 3 times | 12 in 36 3 times |
| 6 in 24 4 times | 9 in 36 4 times | 12 in 48 4 times |
| 6 in 30 5 times | 9 in 45 5 times | 12 in 60 5 times |
| 6 in 36 6 times | 9 in 54 6 times | 12 in 72 6 times |
| 6 in 42 7 times | 9 in 63 7 times | 12 in 84 7 times |
| 6 in 48 8 times | 9 in 72 8 times | 12 in 96 8 times |
| 6 in 54 9 times | 9 in 81 9 times | 12 in 108 9 times |
| 7 in 7 1 time | 10 in 10 1 time | $25 \div 5$ or $\frac{25}{5} =$ |
| 7 in 14 2 times | 10 in 20 2 times | $36 \div 9$ or $\frac{36}{9} =$ |
| 7 in 21 3 times | 10 in 30 3 times | $72 \div 8$ or $\frac{72}{8} =$ |
| 7 in 28 4 times | 10 in 40 4 times | $54 \div 9$ or $\frac{54}{9} =$ |
| 7 in 35 5 times | 10 in 50 5 times | $60 \div 5$ or $\frac{60}{5} =$ |
| 7 in 42 6 times | 10 in 60 6 times | $96 \div 12$ or $\frac{96}{12} =$ |
| 7 in 49 7 times | 10 in 70 7 times | $108 \div 12$ or $\frac{108}{12} =$ |
| 7 in 56 8 times | 10 in 80 8 times | |
| 7 in 63 9 times | 10 in 90 9 times | |

1. Divide 86 by 2.

OPERATION.

Divisor. Dividend.
2)86
43 quotient.

Place the divisor on the left of the dividend, draw a curved line between them, and a straight line under the dividend.

Now, there are 8 tens and 6 units to be divided by 2. We say, 2 in 8, 4 times, which being 4 tens we write the 4 under the tens. We then say, 2 in 6, 3 times, which are three units, and must be written under the 6. The quotient therefore, is 4 tens and 3 units, or 43.

Q. When you divide 8 tens by 2, is the quotient tens or units? When 6 units are divided by 2, what is the quotient?

2. Divide 729 by 3.

| OPERATION. |
| --- |
| 3)729 |
| 243 |

In this example there are 7 hundreds 2 tens and 9 units, all to be divided by 3. Now, we say 3 in 7, 2 times and 1 over. Set down the 2, which is hundreds, under the 7. But of the 7 hundreds there is 1 hundred or 10 tens not yet divided. We put the 10 tens with the 2 tens, making 12 tens, and then say, 3 in 12, 4 times, and write the 4 in the quotient, in the ten's place; then say 3 in 9, 3 times. The quotient therefore, is 243.

Q. When the 7 hundreds are divided by 3, of what denomination is the quotient? To how many tens is the undivided hundred equal? When the 12 tens are divided by 3, what is the quotient? When the 9 units are divided by 3, what is the quotient?

3. Divide 729 by 9.

| OPERATION. |
| --- |
| 9)729 |
| 81 |

In this example, we say, 9 in 7 we cannot, but 9 in 72, 8 times, which are 8 tens: then, 9 in 9, 1 time.

The quotient is therefore 81.

4. Divide 8040 by 8.

| OPERATION. |
| --- |
| 8)8040 |
| 1005 |

In this example, we say 8 in 8, 1 time, and set 1 in the quotient. We then say, 8 in 0, 0 times, and set the 0 in the quotient: then say, 8 in 4, 0 times, and set the 0 in the quotient: then say, 8 in 40, 5 times, and set the 5 in the quotient. The true quotient is therefore 1005.

§ **30**. It may be remarked that any number contains 1 as many times as there are units in the number, or that *if any number be divided by* 1, *the quotient will be equal to the number itself.*

Q. How many times will any number contain 1? If any number be divided by 1, what is the quotient?

CASE I.

§ **31**. Short Division, or when the divisor does not exceed 12.

RULE.

I. *Set down the divisor on the left of the dividend, draw a curved line between them, and a straight line under the dividend.*

II. *Find how often the divisor is contained in the left hand figure or figures of the dividend, and place the figure so found under the straight line, for the first figure of the quotient.*

III. *If there is no remainder, divide the next figure of the dividend for the next figure of the quotient. But when there is a remainder consider it as tens, to which add the next figure of the dividend, regarded as units, and divide this sum for the next figure of the quotient, and do the same for each of the figures of the dividend.*

IV. *When any of the figures, or sums, that are to be divided, is less than the divisor, set down 0 in the quotient, and to such number regarded as tens, add the next figure of the dividend considered as units, and divide the sum for the next figure of the quotient.*

EXAMPLES.

1. Let it be required to divide 36458 by 5.

OPERATION.

5)36458

7291—3 remain.

In this example, we find the quotient to be 7291 and a remainder 3. This 3 ought in fact to be divided by the divisor 5; but the division cannot be effected, since 3 does not contain 5. The division must then be expressed by placing 5 under the 3, thus, $\frac{3}{5}$. The true quotient, therefore, is 7291$\frac{3}{5}$, which is read, seven thousand two hundred and ninety one, and *three divided by five*. Therefore,

*When there is a remainder after the division, it may be written after the quotient, and the divisor placed under it.*

*Q.* What is short division? How do you set down the numbers to be divided? How do you divide? Repeat the rule. If there is a remainder after division, how may it be written?

EXAMPLES.

| (1.) | (2.) | (3.) |
|---|---|---|
| 8)75890496 | 7)3505614 | 6)95040522 |
| 9486312 | 500802 | 15840087 |

4. Divide 6794108 by 3. *Ans.* 2264702$\frac{2}{3}$.
5. Divide 21090431 by 9. *Ans.* 2343381$\frac{2}{9}$.
6. Divide 2345678964 by 6. *Ans.* 390946494.
7. Divide 570196382 by 12. *Ans.* 47516365$\frac{2}{12}$.
8. Divide 67897634 by 9. *Ans.* 7544181–5 remain.
9. Divide 75436298 by 12. *Ans.* ————2 remain.

### CASE II.

§ **32**. Long Division, or when the divisor contains several figures.

### RULE.

I. *Set down the divisor on the left of the dividend draw a curved line between them, and also a curved line on the right of the dividend.*

II. *Note the fewest figures of the dividend, counted from the left hand, that will contain the divisor; find how often they contain it, and set the figure in the quotient.*

III. *Multiply the whole divisor by this figure; set the product under the first figures of the dividend, and subtract it from them.*

IV. *To the remainder annex the next figure of the dividend, then find how often the divisor is contained in this new number, and set the figure in the quotient.*

V. *Multiply the whole divisor by the last figure of the quotient, and subtract the product from the last number containing the divisor. To the remainder annex the next figure of the dividend, and find the figures of the quotient in the same way, till all the figures of the dividend are brought down.*

Note 1. There are four numbers in division. First, the the dividend; second, the divisor; third, the quotient; fourth, the remainder.

Note 2. There are five operations in division. First, to write down the numbers; second, find how many times: third, multiply; fourth, subtract; fifth, bring down.

Q. How do you set down the numbers for division? What do you do next? What do you do next? What is the next step? How many numbers are there in division? What are they? How many operations are there in division? Name them.

### EXAMPLES.

1. Divide 11772 by 327,

```
OPERATION.
Divisor.  Dividend.  Quotient.
327)11772(36
     981
     ----
     1962
     1962
     ----
     0000
```

Having set down the divisor on the left of the dividend, it is seen that 327 is not contained in 117; but by observing that 3 is contained in 11, 3 times and something over, we conclude that the divisor is contained at least 3 times in the first *four* figures of the dividend.

Set down the 3 in the quotient, and multiplying the divisor by it; we thus get 981 which being less than 1177, the quotient figure is not too great: we subtract 981 from the first four figures of the dividend, and find a remainder 196, which being less than the divisor, the quotient figure is not too small.

Annex to this remainder the next figure 2, of the dividend.

As 3 is contained in 19, 6 times, we conclude that the divisor is contained in 1962 as many as 6 times. Setting down 6 in the quotient and multiplying the divisor by it, we find the product to be 1962. Therefore the entire quotient is 36, or the divisor is contained 36 times in the dividend.

NOTE 1. After multiplying by the quotient figure, if any one of the products is greater than the number supposed to contain the divisor, the quotient figure is too large, and must be diminished.

NOTE 2. When any one of the remainders is greater than the divisor, the quotient figure is too small, and must be increased by at least 1.

*Q.* If any one of the products is too large, what do you do? If any one of the remainders is greater than the divisor, what do you do?

2. Divide 2756 by 26.

```
OPERATION.
26)2756(106
   26
   ---
    156
    156
    ---
```

We first say, 26 in 27 once, and place 1 in the quotient. Multiplying by 1, subtracting, and bringing down the 5, we say 26 in 15, 0 times, and place the 0 in the quotient. Bringing down the 6, we find that the divisor is contained in 156, 6 times.

NOTE. If after having annexed the figure from the dividend to any one of the remainders, the number is less than the divisor, the quotient figure is 0, which being written in the quotient, annex the next figure of the dividend and divide as before

## DEMONSTRATION OF THE RULE OF DIVISION.

§ **33.** If 6 simple units be divided by 3, the quotient will be 2. If 6 units of the 2d order, or 60, be divided by 3, the quotient will be 2 tens, or 2 units of the 2d order. If 9 hundreds, or 9 units of the 3rd order be divided by 3, the quotient will be 3 hundreds, or 3 units of the 3d order. So, in general,

*If units of any order be divided by simple units, the units of the quotient will be of the same order as those of the dividend.*

Let us suppose, as an example, that it were required to divide 11772 by 327.

OPERATION.

```
327)11772(36
     981
     ----
     1962
     1962
     ----
```

We first consider, as we have a right to do, that 11772 is made up of 1177 tens and 2 units. We then divide the tens by the divisor 327, and find 3 tens for the quotient, by which we multiply the divisor and subtract the product from 1177, leaving a remainder of 196 tens. To this number we bring down the 2 units, making 1962 units. This number contains the divisor 6 times: that is, 6 unit's times.

When the unit of the first number which contains the divisor is of the 3d order, or 100, there will be 3 figures in the quotient; when it is of the 4th order there will be 4, &c.

Hence, the quotient found according to the rule, expresses the number of times which the dividend contains the divisor, and consequently is the true quotient.

Q. When the divisor is contained in simple units, what units will the quotient figure express? When the divisor is contained in tens, what units will the quotient figure express? When it is contained in hundreds? In thousands?

### PROOF OF DIVISION.

§ **34**. Multiply the divisor by the quotient and add in the remainder, when there is one: the sum should be equal to the dividend.

### EXAMPLES.

1. Divide 67289 by 261.

In this example, we find a quotient of 257 and a remainder of 212, which being less than the divisor will not contain it.

OPERATION.

```
261)67289(257
    522
    ----
    1508
    1305
    ----
     2039
     1827
     ----
      212 Rem.
```

PROOF.

```
  261 Divisor.
  257 Quotient.
 ----
 1827
1305
522
  212 Remainder.
-----
67289=the dividend:  Hence, the work is right.
```

2. Divide 119836687 by 39407.

OPERATION.

```
39407)119836687(3041
      118221
      ------
        161568
        157628
        ------
          39407
          39407
```

PROOF.

```
    39407 Divisor.
     3041 Quotient.
    -----
    39407
  157628
118221
---------
119836687 Dividend.
```

**Q. How do you prove division?**

### PROOF OF MULTIPLICATION.

§ **35**. When two numbers are multiplied together the multiplicand and multiplier are both factors of the product; and if the product be divided by one of the factors, the quotient will be the other factor. Hence, *if the product of two numbers be divided by the multiplicand, the quotient will be the multiplier; or, if it be divided by the multiplier, the quotient will be the multiplicand.*

**Q.** If two numbers are multiplied together, what are the factors of the product? If the product be divided by one of the factors, what will the quotient be? How do you prove multiplication?

### EXAMPLES.

| | |
|---|---|
| 3679 Multiplicand | 3679)1203033(327 |
| 327 Multiplier | 11037 |
| 25753 | 9933 |
| 7358 | 7358 |
| 11037 | 25753 |
| 1203033 Product. | 25753 |
| | 00000 |

2. The multiplicand is 61835720, the product 8162315040: what is the multiplier? *Ans* 132.

3. The multiplier is 270000, the product 1315170000000: what is the multiplicand? *Ans.* 4871000.

4. The product is 68959488, the multiplier 96: what is multiplicand? *Ans.* 718328.

5. The multiplier is 1440, the product 10264849920: what is the multiplicand? *Ans.* 7128368.

6. The product is 6242102428164, the multiplicand 6795634: what is the multiplier? *Ans.* 918546.

7. Divide 7210473 by 37. *Ans.* 194877–24 rem.

8. Divide 147735 by 45. *Ans.* 3283.

9. Divide 937387 by 54. *Ans.* 17359–1 rem.

10. Divide 145260 by 108. *Ans.* 1345.

11. Divide 79165238 by 238. *Ans.* $332627\frac{12}{238}$.

12. Divide 62015735 by 7803. *Ans.* $7947\frac{5294}{7803}$.

13. Divide 74855092410 by 949998. *Ans.* 78795.

14. Divide 47254149 by 4674. *Ans.* ——9 rem.

15. Divide 119184669 by 38473. *Ans.* ——33788 rem.

16. Divide 280208122081 by 912314. *Ans.* ——121 rem.

17. Divide 293839455936 by 8405. *Ans.* ——346 rem.

18. Divide 4637064283 by 57606. *Ans.* ——11707 rem.

19. Divide 352107193214 by 210472.

*Ans.* ——165534 rem.

20. Divide 5580011726061767724 by 2708630425.

*Ans.* ——24 rem.

## CONTRACTIONS IN DIVISION.

### CASE I.

§ **36**. When the divisor is a composite number.

### RULE.

*Divide the dividend by one of the factors of the divisor, and then divide the quotient thus arising by the other factor: the last quotient will be the one sought.*

### EXAMPLES.

Let it be required to divide 1407 dollars equally among 21 men. Here the factors of the divisor are 7 and 3.

OPERATION.

```
7)1407
 3)201  1st quotient.
    67  quotient sought.
```

Let the 1407 dollars be first divided equally among 7 men. Each share will be 201 dollars. Let *each one* of the 7 men divide his share into 3 equal parts, each one of the three equal parts will be 67 dollars, and the whole number of parts will be 21; therefore the true quotient is found by dividing continually by the factors.

2. Divide 18576 by $48=4\times 12$. *Ans.* 387.
3. Divide 9576 by $72=9\times 8$. *Ans.* 133.
4. Divide 19296 by $96=12\times 8$. *Ans.* 201.

§ **37**. It sometimes happens that there are remainders after division, for which we have the following

### RULE.

*The first remainder, if there be one, forms a part of the true remainder. The product of the second remainder, if there be one, by the first divisor, forms a second part. Either of these parts when the other does not exist, forms the true remainder, and their sum makes the true remainder when they both exist together.*

### EXAMPLES.

1. What is the quotient of 751 grapes, divided by 16?

$$4\times4=16\left\{\begin{array}{l}4)751\\4)\overline{187}\ \ldots\ 3\\\quad\overline{46}\ \ldots\ 3\times4=12\end{array}\right.$$

3

15 the true remainder.

*Ans.* $46\frac{15}{16}$.

### DEMONSTRATION OF THE RULE.

In 751 grapes there are 187 sets, (say bunches,) with 4 grapes or units in each bunch, and 3 units over. In the 187 bunches there are 46 piles, 4 bunches in a pile, and 3 bunches over. But there are 4 grapes in each bunch; therefore, the number of grapes in the 3 bunches is equal to $4\times3=12$, to which add 3, the grapes of the first remainder, and we have the entire remainder 15.

2. Divide 4967 by 32.

$$4\times8=32\left\{\begin{array}{l}4)4967\\8)\overline{1241}\ \ldots\ 3\text{, 1st remainder}\\\quad\overline{155}\ \ldots\ 1\times4+3=7\text{ the true remainder.}\end{array}\right.$$

*Ans.* $155\frac{7}{32}$.

3. Divide 956789 by $7\times8=56$. *Ans.* $17085\frac{29}{56}$.
4. Divide 4870029 by $8\times9=72$. *Ans.* $67639\frac{21}{72}$.
5. Divide 674201 by $10\times11=110$. *Ans.* $6129\frac{11}{110}$.
6. Divide 445767 by $12\times12=144$. *Ans.* $3095\frac{87}{144}$.

Q. What is a composite number? (See § 27, page 50). How do you divide when the divisor is a composite number? When there is a remainder, how do you find the true remainder.

### CASE II.

**§ 38.** When the divisor is 10, 100, 1000, &c.

### RULE.

I. *Cut off from the right hand of the dividend as many figures as there are 0's in the divisor.*

II. *The left hand figures of the dividend will express the quotient, and the figures cut off the remainder.*

EXAMPLES.

1. Divide 3256 by 100.

In this example there are two 0's in the divisor, therefore, there are two figures cut off from the right hand of the dividend, and the quotient is 32, and 56÷100.

OPERATION.
100|32|56
*Ans.* $32\frac{56}{100}$.

DEMONSTRATION OF THE RULE.

The quotient ought to be 10, 100, 1000, &c., times less than the dividend. But the same figure is 10, 100, 1000, &c., times greater or less in value, according to its distance from the unit's place. By cutting off figures from the right hand, the unit's place is removed to the left, and consequently the dividend is diminished 10, 100, 1000, &c., times, according as you cut off 1, 2, 3, &c., figures.

2. Divide 49763 by 10. *Ans.* $4976\frac{3}{10}$.

3. Divide 7641200 by 100. *Ans.* 76412.

4. Divide 496321 by 1000. *Ans.* $496\frac{321}{1000}$.

CASE III.

§ **39**. When there are ciphers on the right of the divisor.

RULE.

I. *Cut off the ciphers by a line, and cut off the same number of figures from the right of the dividend.*

II. *Divide the remaining figures of the dividend by the significant figures of the divisor, and annex to the remainder, if there be one, the figures cut off from the dividend: this will form the true remainder.*

EXAMPLES.

1. Divide 67389 by 700.

In this example we strike off the 89, and then find that 7 is contained in the remaining figures, 96 times, with a remainder of 1; to this we annex 89, forming the remainder 189: to the quotient 96 we annex 189 divided by 700 for the entire quotient.

OPERATION.
7|00)673|89
96 . . . 1 remains.
189 true remain.
*Ans.* $96\frac{189}{700}$.

### DEMONSTRATION OF THE RULE.

The number $700 = 100 \times 7$. Hence it is a composite number of which the factors are 100 and 7.

In striking off the two figures 89, from the right of the dividend, we divide it by 100; we then divide the 673 by the other factor 7. We then multiply the remainder 1 by 100 and add 89 to the product, giving 189 for the true remainder, (see § **37**.)

2. Divide 8749632 by 37000.

```
37|000)8749|632(236
       74
       ---
       134
       111
       ---
        239
        222
        ---
         17
```

*Ans*. $236\frac{17632}{37000}$.

3. Divide 986327 by 210000. *Ans*. $4\frac{146327}{210000}$.

4. Divide 876000 by 6000. *Ans*. 146.

5. Divide 36599503 by 400700. *Ans*. $91\frac{135803}{400700}$.

**Q. How do you divide by 10, 100, 1000, &c.? (see § 38). Which part is the quotient? Which part is the remainder? When there are ciphers on the right of the divisor, how do you form the true remainder?**

### APPLICATIONS IN DIVISION.

1. Divide 80 dollars equally among four men.

Here the 80 dollars is to be divided into 4 equal parts, and the quotient 20 dollars expresses the value of one of the equal parts.

OPERATION.

```
4)80
  20 dollars.
```

2. Four persons buy a lottery ticket; it draws a prize of 10000 dollars: what is each one's share?

*Ans*. 2500 *dollars*.

3. A person dying leaves an estate of 4500 dollars to be divided equally among 5 children: what is each one's share? *Ans*. 900 *dollars*.

4. There are 1560 eggs to be packed in 24 baskets: how many eggs will be put in each basket? *Ans.* 65.

5. What number must be multiplied by 124 to produce 40796? *Ans.* 329.

6. How many times can 24 be subtracted from 1416? *Ans.* 59.

7. The sum of 19125 dollars is to be distributed among a certain number of men, each is to receive 425 dollars: how many men are to receive the money? *Ans.* 45.

8. By the census of 1830 the whole population of the 24 States was 12,840,540: if each one had contained an equal number of inhabitants, how many would there have been in each state? *Ans.* 535,022$\frac{12}{24}$.

9. If a man walks 12775 miles in a year, or 365 days, how far does he walk each day? *Ans.* 35 *miles.*

10. A farmer sells a drove of sheep for 2 dollars a head, and receives 1250 dollars: how many sheep did he sell? *Ans.* 625.

11. It is computed that the distance to the sun is 95,000,000 of miles, and that light is 8 minutes travelling from the sun to the earth: how many miles does it travel per minute? *Ans.* 11875000.

12. By the census of 1830 it appeared that the City of New York contained 207020 inhabitants; allowing 5 to each house, how many houses were there in the city at that time? *Ans.* 41404.

13. A merchant has 5100 pounds of tea, and wishes to pack it in 60 chests: how many pounds must he put in each chest? *Ans.* 85.

14. A person goes to a store and buys a piece of cloth containing 36 yards, for which he pays 288 dollars: how much does he pay per yard? *Ans.* 8 *dollars.*

15. There are 7 days in a week: how many weeks in a year of 365? *Ans.* 52 *weeks* and 1 *day* over.

16. There are 24 hours in a day: how many days in 2040 hours? *Ans.* 85 *hours.*

17. Twenty-three persons dined together, their bill was 92 dollars: how much had each one to pay? *Ans.* 4 *dollars.*

## GENERAL REMARKS.

§ **40**. Numeration, Addition, Subtraction, Multiplication, and Division, are called the five ground rules of Arithmetic.

Q. How many principal rules are there in Arithmetic? What are they? Can Multiplication be performed by Addition? Can Division be performed by Subtraction? By how many rules, then, may all the operations in Arithmetic be performed?

§ **41**. The preceding rules furnish answers to the following questions.

*Ques.* 1. When the cost of each one of several things is given, how do you find their entire cost?

*Ans.* Add the costs of the several things together, the sum will be the entire cost.

What is the entire cost of a bag of coffee at 6 dollars, a chest of tea at 4 dollars, a box of raisins at 2 dollars, and a barrel of sugar at 12 dollars? *Ans.* 24 *dollars.*

*Q.* 2. When you have two unequal numbers, how do you find their difference?

*A.* By subtracting the less from the greater.

*Q.* 3. When the subtrahend and remainder are given, or known, how do you find the minuend?

*A.* By adding the remainder and subtrahend together. Hence the following principles.

1st. *If the sum of two numbers be diminished by one of them, the remainder will be the other number.*

2d. *The less of two numbers added to their difference, will give the greater.*

The sum of two numbers is 56, one of the numbers is 12: what is the other? *Ans.* 44.

The less of two numbers is 25, and their difference 30: what is the greater? *Ans.* 55.

The less of two numbers is 35, and their difference 35: what is the greater? *Ans.* 70.

*Q.* 4. When you have the cost of a single thing, how will you find the entire cost of any number of things at the same rate?

*A.* Multiply the cost of the single thing by the number of things.

What is the cost of 35 pears at 2 cents each? What is the cost of 45 yards of cloth at 3 dollars per yard?

*Q.* 5. When you know the number of things, and their entire cost, how do you find the cost of a single thing of the same kind?

*A.* Divide the entire cost by the number of things, the quotient will be the cost of a single thing.

If 60 oranges cost 360 cents, how much do they cost apiece? If 40 yards of cloth cost 200 dollars, how much is it a yard?

### APPLICATIONS IN THE PRECEDING RULES.

1. A Farmer sells a yoke of oxen for 90 dollars, 3 cows for 25 dollars each, 9 calves for 4 dollars each, and 65 sheep at 3 dollars a head. How much did he receive for them all? *Ans.* 396 *dollars.*

2. The sum of two numbers is 365, one of the numbers is 221; what is the other number? *Ans.* 144.

3. The difference of two numbers is 95, the less number is 327; what is the greater number? *Ans.* 422.

4. A farmer sells 4 tons of hay at 12 dollars per ton, 80 bushels of wheat at 1 dollar per bushel, and takes in part payment a horse worth 65 dollars, a waggon worth 40 dollars, and the rest in cash. How much money did he receive? *Ans.* 23 *dollars.*

5. A farmer has 14 calves worth 4 dollars each, 40 sheep worth 3 dollars each; he gives them all for a horse worth 150 dollars: does he make or lose by the bargain? *Ans.* He loses 26 *dollars.*

6. The product of two numbers is 51679680, and one of the factors is 615: what is the other factor? *Ans.* 84032.

7. When the divisor is 67941, and the quotient 30620, what is the dividend? *Ans.* 2080353420.

8. When the dividend is 1213193, the quotient 37, what is the divisor? *Ans.* 32789.

9. A piece of cloth containing 65 yards costs 455 dollars: what does it cost per yard? *Ans.* 7 *dollars.*

10. A man has 6 children, all of whom are married, and each has four children; two of these grand-children are

married, and each has one child: how many children, grand-children, and great grand-children are there?

*Ans.* 32.

11. The distance around the earth is computed to be about 25000 miles: how long would it take a man to travel it, supposing him to travel at the rate of 35 miles a day? *Ans.* $714\frac{10}{35}$ *days.*

12. The earth moves around the sun at the rate of 68000 miles an hour: how many miles does it travel in a day, and how many in a year? *Ans.* { 1632000 *in a day.* 595680000 *in a year.* }

13. A farmer purchased a farm for which he paid 18050 dollars. He sold 50 acres for 60 dollars an acre, and the remainder stood him in 50 dollars per acre: how much land did he purchase? *Ans.* 351 *acres.*

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## OF FRACTIONS.

§ **42.** The unit 1 represents an entire thing; as 1 apple, 1 chair, 1 pound of tea.

If we suppose one thing, as one apple, or one pound of tea, to be divided into two equal parts, each part is called *one half* of the thing.

If the unit be divided into 3 equal parts, each part is called *one third.*

If the unit be divided into 4 equal parts, each part is called *one fourth.*

If the unit be divided into 12 equal parts, each part is called *one twelfth;* and when it is divided into any number of equal parts, we have a similar expression for each of the parts.

The equal parts of a thing are expressed thus:

| | | | |
|---|---|---|---|
| $\frac{1}{2}$ is read | one half. | $\frac{1}{7}$ is read | one seventh. |
| $\frac{1}{3}$ - - | one third. | $\frac{1}{8}$ - - | one eighth. |
| $\frac{1}{4}$ - - | one fourth. | $\frac{1}{10}$ - - | one tenth. |
| $\frac{1}{5}$ - - | one fifth. | $\frac{1}{15}$ - - | one fifteenth. |
| $\frac{1}{6}$ - - | one sixth. | $\frac{1}{50}$ - - | one fiftieth. |

The $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., are called *fractions.*

**Q. What does the unit 1 represent? If we divide it into two equal parts, what is each part called? If it be divided into three equal parts, what is each part? Into 4, 5, 6, &c., parts? What are such expressions called?**

§ **43.** Each fraction is made up of two numbers; the number which is written above the line is called the *numerator;* and the one below it is called the *denominator*, because it gives a denomination or name to the fraction.

For example, in the fraction $\frac{1}{2}$, 1 is the numerator, and 2 the denominator. In the fraction $\frac{1}{3}$, 1 is the numerator, and 3 the denominator.

*The denominator in every fraction shows into how many equal parts the unit or single thing, is divided.* For example, in the fraction $\frac{1}{2}$, the unit is divided into 2 equal parts; in the fraction $\frac{1}{3}$, it is divided into 3 equal parts; in the fraction $\frac{1}{4}$, it is divided into 4 equal parts, &c. In each of the fractions *one* of the equal parts is expressed. But suppose it were required to express 2 of the equal parts, as 2 halves, 2 thirds, 2 fourths, &c.

We should then write,

| | | |
|---|---|---|
| $\frac{2}{2}$ | they are read | two halves. |
| $\frac{2}{3}$ | - - - | two thirds. |
| $\frac{2}{4}$ | - - - | two fourths. |
| $\frac{2}{5}$ | - - - | two fifths, &c. |

If it were required to express three of the equal parts, we should place 3 in the numerator; and generally, *the numerator shows how many of the equal parts are expressed in the fraction.*

For example, three eighths are written,

| | | |
|---|---|---|
| $\frac{3}{8}$ | and read | three eighths. |
| $\frac{4}{9}$ | - - | four ninths. |
| $\frac{6}{13}$ | - - | six thirteenths. |
| $\frac{9}{20}$ | - - | nine twentieths. |

**Q. Of how many numbers is each fraction made up? What is the one above the line called? The one below the line? What does the denominator show? What does the numerator show? In the three eighths, which is the numerator? Which the denominator? Into how many parts is the unit divided? How many parts are expressed? In the fraction nine-twentieths, into how many parts is the unit divided? How many parts are expressed?**

§ 44. When the numerator and denominator are equal, the numerator expresses all the equal parts into which the unit has been divided: therefore, the *value of the fraction is equal to* 1. But if we suppose a second unit, of the same kind, to be divided into the same number of equal parts, those parts may also be expressed in the same fraction with the parts of the first unit. Thus,

$\frac{3}{2}$ is read three halves.
$\frac{7}{4}$ - - seven fourths.
$\frac{16}{5}$ - - sixteen fifths.
$\frac{18}{6}$ - - eighteen sixths.
$\frac{25}{7}$ - - twenty-five sevenths.

The denominator of the first fraction, shows that a unit has been divided into 2 equal parts, and the numerator expresses that three such parts are taken. Now, two of the parts make up one unit, and the remaining part comes from the 2d unit: hence, the *value* of the fraction is $1\frac{1}{2}$; that is, one and one half.

The denominator of the second fraction, shows that a unit has been divided into four equal parts, and the numerator expresses that 7 such parts are taken. Four of the 7 parts come from one unit, and the remaining 3 from a second unit: the *value* of the fraction is therefore equal to to $1\frac{3}{4}$; that is, to one and three-fourths. In the third fraction, the unit has been divided into 5 equal parts, and 16 such parts are taken. Now, since each unit has been divided into 5 parts, 15 of the 16 parts make 3 units, and the remaining part is 1 part of a fourth unit. Therefore, the *value* of the fraction is $3\frac{1}{5}$: that is, three and one fifth. The value of the fourth fraction is three, and of the fifth, three and four-sevenths. From what has been said, we conclude:

1st. *That a fraction is the expression of one or more parts of unity.*

2d. *That the denominator of a fraction shows into how many equal parts the unit or single thing has been divided, and the numerator expresses how many such parts are taken in the fraction.*

3d. *That the value of every fraction is equal to the quotient arising from dividing the numerator by the denominator.*

4th. *When the numerator is less than the denominator, the value of the fraction is less than* 1.

5th. *When the numerator is equal to the denominator, the value of the fraction is equal to* 1.

6th. *When the numerator is greater than the denominator, the value of the fraction is greater than* 1.

Q. When the numerator and denominator are equal, what is the value of the fraction? What is the value of the fraction three halves? Of seven fourths? Of sixteen fifths? Of eighteen sixths? Of twenty-five sevenths? Repeat the six principles? Write the fraction nineteen fortieths:—also, 60 fourteenths—18 fiftieths—16 twentieths—17 thirtieths—41 one thousandths—69 ten thousandths—85 millionths—106 fifths.

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## OF DENOMINATE NUMBERS.

§ 45. Simple numbers express a collection of units of the same kind, without expressing the particular value of the unit. For example, 40 and 55 are simple numbers, and the unit is 1, but it is not expressed whether the unit is 1 apple, 1 pound, or 1 horse.

A DENOMINATE number expresses the *kind* of unit which is considered. For example, 6 dollars is a denominate number, the *unit* 1 dollar being denominated, or named.

When two numbers have the same unit, they are said to be of the same denomination: and when two numbers have different units, they are said to be of different denominations.

For example, 10 dollars and 12 dollars are of the same denomination; but, 8 dollars and 20 cents, express numbers of different denominations, the unit of 8 dollars being 1 dollar, and of 20 cents, 1 cent.

Several numbers of different denominations are often connected together, forming a whole, as 3 dollars 15 cents.

Q. What do simple numbers express? What is a denominate number? What is the unit of 6 dollars? When two numbers have the same unit, what do you say of them? When they have different units? Are 6 dollars and 4 dollars of the same denomination? Arc

**4 dollars and 4 cents? What is the unit of each? Are several numbers of different denominations often connected together? Give an example?**

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## OF FEDERAL MONEY.

§ **46.** Federal money is the currency of the United States. Its denominations, or names, are Eagles, Dollars, Dimes, Cents, and Mills.

The coins of the United States are of gold, silver, and copper, and are of the following denominations.

1. Gold—Eagle, half-eagle, quarter-eagle.

2. Silver—Dollar, half-dollar, quarter-dollar, dime, half-dime.

3. Copper—Cent, half-cent.

If a given quantity of gold or silver, be divided into 24 equal parts, each part is called a *carat*. If any number of carats be mixed with so many equal carats of a less valuable metal, that there be 24 carats in the mixture, then the compound is said to be as many carats fine as it contains carats of the more precious metal, and to contain as much alloy as it contains carats of the baser.

For example, if 20 carats of gold be mixed with 4 of silver, the mixture is called gold of 20 carats fine, and 4 parts alloy. The standard of the gold coin in the United States, is 22 carats of gold, 1 of silver, and 1 of copper.

The standard for silver coins is 1489 parts of pure silver, to 179 of pure copper.

The copper coins are of pure copper.

**Q. What is the currency of the United States? What are its denominations? What are the coins of the United States? Which gold? Which silver? Which copper? What do you understand by gold 20 carats fine? What is the standard of the gold coin? What of the silver coin? What of the copper?**

### TABLE OF FEDERAL MONEY.

| | | | |
|---|---|---|---|
| 10 Mills marked (*m*) | make | 1 Cent, marked | *ct.* |
| 10 Cents | - - - - - - | 1 Dime, - - | *d.* |
| 10 Dimes | - - - - - - | 1 Dollar, - - | $. |
| 10 Dollars | - - - - - - | 1 Eagle, - - | *E.* |

In this table, 10 *units* of either denomination make one unit of the next higher denomination, and this is the same way that simple numbers increase from the right to the left. Therefore,

*The denominations of federal money here expressed may be added, subtracted, multiplied, and divided, by the same rules that have already been given for simple numbers.*

From the table it appears,

1st. *That cents may be changed into mills by annexing a cipher.*

Thus, 8 cents are equal to 80 mills.

2d. *That dollars may be changed into cents by annexing two ciphers, and into mills by annexing three.*

For example, 12 dollars are equal to 1200 cents, or 12000 mills. The reason of these rules is evident, since 10 mills make a cent, 100 cents a dollar, and 1000 mills a dollar.

Q. Repeat the table? How many units of either denomination make one of the next higher? How do simple numbers increase from the right to the left? How may Federal Money be added, subtracted, multiplied, and divided? How may cents be changed into mills? How may dollars be changed into cents? How into mills? To how many cents are 12 dollars equal? To how many mills are they equal? How many cents in 4 dollars? How many in 6 dollars? How many mills in 9 dollars? How many mills in 5 dollars? How many cents in 3 dollars? In 8 dollars? In 7 dollars?

## NUMERATION TABLE FOR FEDERAL MONEY.

| Thousands of dollars | Hundreds of dollars. | Tens of dollars or Eagles. | Dollars. | Tens of cents or dimes. | Cents. | Mills | |
|---|---|---|---|---|---|---|---|
| | | | | | 5 | 7, | is read, 5 cents and 7 mills, or 57 mills. |
| | | | | 1 | 6 | 4, | - - 16 cents and 4 mills, or 164 mills. |
| | | 6 | 2, | 1 | 2 | 0, | - - 62 dollars 12 cents and no mills. |
| | 1 | 2 | 7, | 6 | 2 | 3, | - - 127 dollars, 62 cents and 3 mills. |
| 8 | 9 | 4 | 0, | 0 | 4 | 1, | - - 8940 dollars, 4 cents and 1 mill. |

As dimes are tens of cents the second line may either be read 16 cents and 4 mills, or 1 dime 6 cents and 4 mills. And as the eagles are tens of dollars the third line may be read 62 dollars and 12 cents, or 6 eagles 2 dollars and 12 cents.

Federal Money is generally read in dollars cents and mills.

Q. In numerating Federal Money, what is the figure on the right called? The second? The third? The fourth? How is Federal Money generally read?

## REDUCTION OF FEDERAL MONEY.

§ **47**. REDUCTION of Federal Money consists in changing its denominations without altering its value. It is divided into two parts.

1st. To reduce from a higher denomination to a lower, as from dollars to cents.

2d. To reduce from a lower denomination to a higher, is from mills to dollars.

Q. What is reduction? How many kinds of reduction are there? Name them?

### EXAMPLES.

1. Reduce 25 Eagles 8 dollars 65 dimes and 35 cents, to the denomination of cents.

OPERATION.

| | |
|---|---|
| 25 | Eagles the highest denomination. |
| 10 | dollars make one eagle. |
| 250 | Product in dollars. |
| add 8 | the number in the denomination of dollars. |
| 258 | |
| 10 | the number of dimes in a dollar. |
| 2580 | Product in dimes. |
| add 65 | the number in the denomination of dimes. |
| 2645 | |
| 10 | number of cents in a dime. |
| 26450 | Product in cents. |
| 35 | cents to be added. |
| 26485 | Number of cents in 25 eagles 8 dollars 65 dimes and 35 cents. |

2. In 3 dollars 60 cents and 5 mills; how many mills?

3 dollars =300 cents,
60 cents to be added,
360=3600 mills, to which add the 5 mills.
*Ans.* 3605.

3. In 37 dollars 37 cents 8 mills: how many mills? *Ans.* 37378.

4. In 375 dollars 99 cents 9 mills: how many mills? *Ans.* 375999.

5. How many mills in 67 cents? *Ans.* 670.

6. How many mills in $54? *Ans.* 54000.

7. How many cents in $125? *Ans.* 12500.

8. In $400, how many cents? how many mills?

9. In $375, how many cents? How many mills?

10. How many mills in $4? In $6? In $10,14 cents?

11. How many mills in $40,36 cents 8 mills?

§ 48. As we change dollars into cents by adding two ciphers, and cents into mills by adding one, it follows that, to change mills into dollars cents and mills, we have the following

RULE.

*Cut off the right hand figure for mills, and the figures to the left will be cents. Then cut off the two next figures for cents, and the remaining figures to the left will be dollars.*

The reason of the rule is this: by cutting off the first right hand figure, we in fact, divide by 10, and thus reduce the mills to cents. Then by cutting off the next two figures, we divide by 100; and thus reduce the cents to dollars.

The comma, or separatrix, is generally used to separate the cents from the dollars. It is not usual to place the comma between the cents and mills. Thus, $67,25 6 is read 67 dollars 25 cents and 6 mills.

Q. How do you change mills into cents? How do you change cents into dollars? How do you separate the mills from the cents? How the cents from the dollars?

EXAMPLES.

1. How many dollars cents and mills, are there in 67897 mills? *Ans.* $67,89 7.

2. Set down 104 dollars 69 cents and 8 mills.
*Ans.* \$104,69 8.

3. Set down 4096 dollars 4 cents and 2 mills.
*Ans.* \$4096,04 2.

4. Set down 100 dollars 1 cent and 1 mill.
*Ans.* \$100,01 1.

5. Write down 4 dollars and 6 mills. *Ans.* \$4,00 6.

6. Write down 109 dollars and 1 mill. *Ans.* \$109,00 1.

7. Write down 65 cents and 2 mills. *Ans.* \$0,65 2.

8. Write down 2 mills. *Ans.* \$0,00 2.

9. Reduce 1607 mills, to dollars and cents.
*Ans.* \$1,60 7.

10. Reduce 170464 mills, to dollars. *Ans.* \$170,46 4.

11. Reduce 8674416 mills, to dollars.

12. Reduce 94780900 mills, to dollars.

13. Reduce 74164210 mills to dollars.

The parts of a dollar are sometimes expressed fractionally, as in the following

TABLE

| | |
|---|---|
| \$1 = 100 cents, | $\frac{1}{8}$ of a dollar = $12\frac{1}{2}$ cents, |
| $\frac{1}{2}$ of a dollar = 50 cents, | $\frac{1}{10}$ of a dollar = 10 cents, |
| $\frac{1}{3}$ of a dollar = $33\frac{1}{3}$ cents, | $\frac{1}{16}$ of a dollar = $6\frac{1}{4}$ cents, |
| $\frac{1}{4}$ of a dollar = 25 cents, | $\frac{1}{20}$ of a dollar = 5 cents, |
| $\frac{1}{5}$ of a dollar = 20 cents, | $\frac{1}{2}$ of a cent = 5 mills. |

Q. How many cents in a dollar? In half a dollar? In a third of a dollar? In a fourth of a dollar? In the fifth of a dollar? In the eighth of a dollar? In the tenth of a dollar? In the sixteenth of a dollar? In the twentieth of a dollar? How many mills in half a cent!

## ADDITION OF FEDERAL MONEY.

1. Charles gives $9\frac{1}{2}$ cents for a top, and $3\frac{1}{2}$ cents for 6 quills: how much do they cost him? *Ans.* 13 *cents.*

2. John gives \$$1,37\frac{1}{2}$ for a pair of shoes, 25 cents for a pen-knife, and $12\frac{1}{2}$ cents for a pencil: how much does he pay for all?

We first recollect that half a cent is equal to 5 mills. We then place the mills under each other, the cents under cents, and the dollars under dollars. We then add as in simple numbers.

OPERATION.

| |
|---:|
| $1, 37 5 |
| 25 |
| 12 5 |
| $1, 75 |

3. James gives 50 cents for a dozen oranges, 12½ cents for a dozen apples, and 30 cents for a pound of raisins: how much for all?

OPERATION.

| |
|---:|
| $0, 50 |
| 12 5 |
| 30 |
| $0, 92 5 |

Hence, for the addition of Federal Money, we have the following

RULE.

§ 49. I. *Set down the numbers to be added under one another, so that dollars shall fall under dollars, cents under cents, and mills under mills.*

II. *Then add up the several columns as in simple numbers, and place the separating point in the amount directly under those in the columns.*

Q. How do you set down Federal Money for addition? How do you add up the columns? How do you place the separating point?

EXAMPLES.

1. Add $67, 21 4, $10, 04 9, $6, 04 1, $0, 27 1, together.

| (1.) | (2.) | (3.) |
|---:|---:|---:|
| $ cts. m. | $ cts. m. | $ cts. m. |
| 67, 21 4 | 59, 31 6 | 81, 05 3 |
| 10, 04 9 | 87, 42 5 | 67, 41 2 |
| 6, 04 1 | 48, 87 2 | 95, 37 6 |
| 0, 27 1 | 56, 70 8 | 87, 06 4 |
| $83, 57 5 | $252, 32 1 | $330, 90 5 |

| (4.) | (5.) | (6.) |
|---:|---:|---:|
| $375, 02 1 | $ 27, 09 8 | $ 7, 00 9 |
| 2, 09 6 | 325, 59 2 | 0, 01 1 |
| 0, 47 9 | 25, 60 3 | 0, 00 1 |
| 3, 01 2 | 9, 99 9 | 46, 67 9 |
| $380, 60 8 | $388, 29 2 | $53, 70 0 |

APPLICATIONS.

1. A grocer purchased a box of candles for 6 dollars 89 cents; a box of cheese for 25 dollars 4 cents and 3 mills; a keg of raisins for 1 dollar 12½ cents, (or 12 cents and 5 mills;) and a cask of wine for 40 dollars 37 cents 8 mills: what did the whole cost him? *Ans.* $73, 43 6.

2. A farmer purchased a cow for which he paid 30 dollars and 4 mills; a horse for which he paid 104 dollars 60 cents and 1 mill; a wagon for which he paid 85 dollars and nine mills: how much did the whole cost?
*Ans.* $219, 61 4.

3. A man is indebted to A, $630,49; to B, $25; to C, 87½ cents; to D, 4 mills: how much does he owe?
*Ans.* $656, 36 9.

4. Bought 1 gallon of molasses at 28 cents per gallon; a half pound of tea for 78 cents; a piece of flannel for 12 dollars 6 cents and 3 mills; a plough for 8 dollars 1 cent and 1 mill; and a pair of shoes for 1 dollar 20 cents: what did the whole cost? *Ans.* $22, 33 4.

5. Bought 6 pounds of coffee for 1 dollar 12½ cents; a wash-tub for 75 cents 6 mills; a tray for 26 cents 9 mills; a broom for 27 cents; a box of soap for 2 dollars 65 cents 7 mills; a cheese for 2 dollars 87½ cents: what is the whole amount? *Ans.* $7,95 2.

6. What is the entire cost of the following things: viz. 2 gallons of molasses 57 cents; half a pound of tea 37½ cents; 2 yards of broad cloth $3, 37½ cents; 8 yards of flannel $9, 87 5; two skeins of silk 12½ cents, and 4 sticks of twist 8½ cents.

SUBTRACTION OF FEDERAL MONEY.

1. John gives 9½ cents for a pencil, and 8 cents for a top: how much more does he give for the pencil than top? *Ans.* $0, 01 5.

2. A man buys a cow for $26,37, and a calf for $4,50: how much more does he pay for the cow than calf?

| | OPERATION. |
|---|---|
| We set down the numbers as in addition, and then subtract them as in simple numbers. | \$26, 37<br>4, 50<br>\$21, 87 |

Hence, for subtraction of Federal Money, we have the following

RULE.

§ 50. *Place the lesser number under the greater so that the commas, or separating points, shall fall directly under each other; then subtract as in simple numbers, and place the separating point in the remainder directly under those above.*

Q. How do you set down the numbers for subtraction? How do you subtract them? How do you place the separating point in the remainder?

EXAMPLES.

| | (1.) | | (2.) |
|---|---|---|---|
| From | \$204, 67 9 | From | \$8976, 40 0 |
| Take | 98, 71 4 | Take | 610, 09 8 |
| Remainder | \$105, 96 5 | Remainder | \$8366, 30 2 |

| (3.) | (4.) | (5.) |
|---|---|---|
| \$620, 00 0 | \$327, 00 1 | \$2349, |
| 19, 02 1 | 2, 09 0 | 29, 33 |
| \$600, 97 9 | \$324, 91 1 | \$2319, 67 |

6. What is the difference between \$6 and 1 mill? Between \$9,75 and 8 mills? Between 75 cents and 6 mills? Between \$87, 35 4 and 9 mills?

7. From \$107, 00 3 take \$0, 47 9. *Ans.* \$106, 52 4.

APPLICATIONS.

1. A man's income is \$3000 a year; he spends \$187,50: how much does he lay up? *Ans.* \$2812,50.

2. A man purchased a yoke of oxen for \$78, and a cow for \$26, 00 3: how much more did he pay for the oxen than for the cow? *Ans.* \$51, 99 7.

3. A man buys a horse for \$97,50, and gives a hundred dollar bill: how much money ought he to receive back? *Ans.* \$2, 50.

4*

4. How much must be added to \$60, 03 9 to make the sum \$1005,40? *Ans.* \$945, 36 1.

5. A man sold his house for \$3005, this sum being \$98, 03 9 more than he gave for it: what did it cost him? *Ans.* \$2906, 96 1.

6. A man bought a pair of oxen for \$100, and sold them again for \$75,37½: did he make or lose by the bargain, and how much? *Ans.* He lost \$24, 62 5.

7. A man starts on a journey with \$100; he spends \$87, 57: how much has he left? *Ans.* \$12,43.

8. How much must you add to \$40, 17 3 to make \$100? *Ans.* \$59, 82 7.

9. A man purchased a pair of horses for 450, but finding one of them injured, the seller agreed to deduct \$106, 32 5: what had he to pay? *Ans.* \$343, 67 5.

## MULTIPLICATION OF FEDERAL MONEY.

1. John gives 3 cents a piece for 6 oranges: how much do they cost him? *Ans.* 18 *cents.*

2. John buys 6 pair of stockings, for which he pays 25 cents a pair: how much do they cost him?

3. A farmer sells 8 sheep for \$1,25 each: how much does he receive for them?

We multiply the cost of one sheep by the number of sheep, and the product is the entire cost?

OPERATION.

| |
|---|
| \$1,25 |
| 8 |
| \$10,00 |

Hence, for the multiplication of Federal Money by a simple number, we have the following

### RULE.

§ **51**. *Multiply as in simple numbers, and the product will be the answer in the lowest denomination mentioned in the multiplicand; then reduce the product to dollars and cents.*

Q. How do you multiply Federal Money? What will be the denomination of the product? How will you then reduce it to dollars and cents?

## EXAMPLES.

1. Multiply 375 dollars 28 cents and 2 mills, by 8; also, $475,87 by 9.

| OPERATION. | (2.) |
| ---: | ---: |
| $375, 28 2 | $475, 87 |
| 8 | 9 |
| Product $3002, 25 6 | Product $4282, 83 |

| (3.) | (4.) | (5.) |
| ---: | ---: | ---: |
| $3, 00 4 | $89, 07 9 | $81, 99 2 |
| 12 | 7 | 6 |
| $36, 04 8 | $623, 55 3 | $491, 95 2 |

## APPLICATIONS.

1. What will 55 yards of cloth come to at 37 cents per yard? *Ans.* $20, 35.

2. What will 300 bushels of wheat come to at $1,25 per bushel? *Ans.* $375.

3. What will 85 pounds of tea come to at 1 dollar 37½ cents per pound?

In this example we first consider that ½ of a cent is equal to 5 mills. Then as $1,37 5 contains more figures than 85, we multiply by the 85, knowing that the product will be the same which ever number be made the multiplier. The product 116875 is in mills, which is reduced to dollars and cents as before.

| OPERATION. |
| ---: |
| 1375 |
| 85 |
| 6875 |
| 11000 |
| 116875 |
| *Ans.* $116,875 |

4. What will a firkin of butter containing 90 pounds come to at 25½ cents per pound? *Ans.* $22,95.

5. What is the cost of a cask of wine containing 29 gallons, at 2 dollars and 75 cents per gallon? *Ans.* $79,75.

6. A bale of cloths contains 95 pieces, costing 40 dollars 37½ cents each: what is the cost of the whole bale? *Ans.* $3835, 62 5.

7. What is the value of 300 hats at 3 dollars and 25 cents apiece? *Ans.* $975.

8. What is the value of 9704 oranges at $3\frac{1}{2}$ cents each? *Ans.* $339,64.

9. What will be the cost of 356 sheep at $3\frac{1}{4}$ dollars a head? *Ans.* $1157.

10. What will be the cost of 47 barrels of apples at $1\frac{3}{4}$ dollars per barrel? *Ans.* $82,25.

11. What is the value of 6000 bricks at $4,37\frac{1}{2}$ per thousand? *Ans.* $26,25.

## DIVISION OF FEDERAL MONEY.

§ **52**. To divide a sum expressed in dollars, cents, and mills, by a simple number.

### RULE.

I. *If the number to be divided contains dollars cents and mills, divide as in simple numbers, and separate the quotient into dollars cents and mills.*

II. *But if the number to be divided contains only dollars, or dollars and cents, bring it to mills by annexing ciphers: then divide as in simple numbers, and separate the quotient as before.*

Q. How do you divide in Federal Money? When the number to be divided contains only dollars, how do you divide?

### EXAMPLES.

1. Divide $4, 62 4 by 4; also 87, 25 6 by 5.

OPERATION.

4) $4, 62 4
$1, 15 6

OPERATION.

5) $87, 25 6
$17, 45 $1\frac{1}{5}$

2. Divide $37 by 8.

In this example we first reduce the $37 to mills by annexing three ciphers. The quotient will then be mills, and can be reduced to dollars and cents, as before.

OPERATION.

8) $37, 00 0
$ 4, 62 5

3. Divide $56, 17 by 16.

In this example we find the quotient to be 3 dollars 51 cents, and a remainder of 10 mills, which being divided by 16 gives $\frac{10}{16}$ of a mill.

OPERATION.

16)$56, 17 0($3, 51 $\frac{10}{16}$
48
81
80
17
16
10

The answer is always sufficiently exact when it is true within 1 mill, and therefore the remainder in mills may always be neglected. But in common business the quotient figure in mills is neglected. When however, such quotient figure is greater than 5, one may be added to the cents. The sign + is added in the examples, to show that the division may be continued.

Q. When is the answer sufficiently exact? In common business are the mills considered? When they exceed five, what do you do? How do you denote that the division may be continued?

4. Divide $495, 70 4 by 129. *Ans.* $3, 84 +.
5. Divide $12 into 200 equal parts. *Ans.* $0,06.
6. Divide $400 into 600 equal parts. *Ans.* $0, 66 +.
7. Divide $857 into 51 equal parts. *Ans.* $16, 80 +.
8. Divide $6578,95 in 157 equal parts. *Ans.* $41, 90 4 +.
9. Divide $248,54 by 125. *Ans.* $1, 98 8 +.
10. Divide $100 by 33. *Ans.* 3, 03 0 +.

APPLICATIONS.

1. A man bought a piece of cloth containing 72 yards, for which he paid $252: what did he pay per yard? *Ans.* $3,50.

2. If $600 be divided equally among 26 persons: what will be each one's share? *Ans.* $23,07 +.

3. Divide $18000 into 40 equal parts: what is the value of each part? *Ans.* $450.

4. Divide $3769, 25 into 50 equal parts: what is one part? *Ans.* $75,38 +.

5. A farmer purchased a farm containing 725 acres, for which he paid $18306,25: what did it cost him per acre? *Ans.* $25,25.

6. A merchant buys 15 bales of goods at auction, for which he pays $1000: what do they cost him per bale? *Ans.* $66,66 6 +.

7. A drover pays $1250 for 500 sheep: what shall he sell them for apiece, that he may neither make nor lose by the bargain? *Ans.* $2,50.

8. The dairy of a farmer produces $600, and he has 25 cows: how much does he make by each cow? *Ans.* $24.

9. A farmer receives $840 for the wool of 1400 sheep: how much does each sheep produce him? *Ans.* $0,60.

### APPLICATIONS IN THE FOUR PRECEDING RULES.

1. A farmer sold a yoke of oxen for $80,75; 6 cows for $29 each; 30 sheep at $2,50 a head; and 3 colts, one for $25, the other two for $30 apiece: what did he receive for the whole lot? *Ans.* $414,75.

2. A merchant buys 6 bales of goods, each containing 20 pieces of broadcloth, and each piece of broadcloth contained 29 yards; the whole cost him $15660: how many yards of cloth did he purchase, and how much did it cost him per yard? *Ans.* { 3480 *yards.* $4,50 *per yard.*

3. A man dies leaving an estate of $33000 to be equally divided among his 4 children, after his wife shall have taken her third. What was the wife's portion, and what the part of each child? *Ans.* { $11000 *wife's part.* $5500 *each child's part.*

4. A person sells 3 cows at $25 each; and a yoke of oxen for $65: he agrees to take in payment 60 sheep: how much do his sheep cost him per head? *Ans.* $2,33 3 +.

5. A person settling with his butcher, finds that he is charged with 126 pounds of beef at 9 cents per pound; 85 pounds of veal at 6 cents per pound; 6 pair of fowls at 37 cents a pair; and three hams at $1,50 each: how much does he owe him? *Ans.* $23,16.

6. A farmer agrees to furnish a merchant 40 bushels of rye at 62 cents per bushel, and to take his pay in coffee at 16 cents a pound: how much coffee will he receive? *Ans.* 155 *pounds.*

7. A farmer bargains with his tailor for a new coat every six months, a new vest every three months, and three pairs of pantaloons a year: the coats to cost $29,50 each, the vests $3 apiece, and the pantaloons $12 a pair: at the end of two years how much did he owe him?

*Ans.* $214.

8. A farmer raises 300 bushels of wheat, for which he receives $1,37½ per bushel; 500 bushels of potatoes at 29 cents a bushel; 1000 bushels of oats at 34 cents a bushel, and 75 tons of hay for which he receives 16 dollars per ton: how much does the whole come to?

*Ans.* $2097,50.

9. A farmer has six ten acre lots in each of which he pastures 6 cows: each cow produces 112 pounds of butter, for which he receives 18½ cents per pound: the expenses of each cow are 5 dollars and a half: how much does he make by his dairy? *Ans.* $547,92.

10. A man lets out 2000 sheep, with the condition that he is to have three fourths of what they produce after deducting the expenses of shearing: they yield 4 pounds of wool ahead, which is sold at 47½ cents per pound. The expense of shearing is one tenth of the whole: what does the owner of the sheep receive? *Ans.* $2565.

11. A man lets out his farm on shares. He is to have half the grain, one third the price of the hay, and one quarter the increase of the live stock. At the end of the time, there have been raised, 500 bushels of wheat worth $1,87½ a bushel, 300 bushels of oats worth 37½ cents a bushel, 250 bushels of corn worth 80 cents a bushel, 65 tons of hay worth $18 a ton; and the increase of the live stock, had been, 5, two years old worth $8 apiece, 8 calves worth $5 apiece, 10 sheep worth $2 apiece, a colt worth $36, and a pair of steers worth $28,50: how much was the owner of the land to receive? *Ans.* $1056,12½.

## BILLS OF PARCELS.

New York, May 1st, 1837.

*Mr. James Spendthrift*

*Bought of Benj. Saveall.*

16 pounds of tea at 85 cents per pound - - -
27 pounds of coffee at 15½ cents per pound - -
15 yards of linen at 66 cents per yard - - -

$27, 68 5.

Rec'd payment, *Benj. Saveall.*

---

Albany, June 2d, 1837.

*Mr. Jacob Johns* *Bought of Gideon Gould.*

36 pounds of sugar at 9½ cents per pound - - -
3 hogsheads of molasses, 63 galls. each at 27 cents a gallon - - - - - - -
5 casks of rice 285 pounds each, at 5 cts. per pound
2 chests of tea 86 pounds each, at 96 cts. per pound

Total cost $290,82.

Rec'd payment, For Gideon Gould,
*Charles Clark.*

---

Hartford, November 21st, 1837.

*Gideon Jones* *Bought of Jacob Thrifty.*

69 Chests of tea at $55,65 per chest - - - -
126 Bags of coffee, 100 pounds each, at 12½ cts. per pound - - - - - - - - - - - -
167 Boxes of raisins at $2,75 per box - - -
800 Bags of almons at $18,50 per bag - - -
9004 Barrels of shad at $7,50 per barrel - - -
60 Barrels of oil 32 gallons each, at $1,08 per gall.

Amount $90277,70.

Received the above in full, *Jacob Thrifty.*

# DENOMINATE NUMBERS.

§ **53**. There are other denominate numbers besides those of Federal Money. For example, 6 yards of cloth is a denominate number, the *unit*, 1 yard of cloth, being denominated, or named.

Two numbers are of the same denomination, when they have the same unit, and of different denominations when they have different units.

For example, 8 feet and 10 feet are of the same denomination, the unit being 1 foot; but 30 feet and 60 yards are of different denominations, the unit of the first being 1 foot, and the unit of the second, 1 yard.

Q. What is a denominate number? (see § 45). What is the unit of 6 yards of cloth? When are two numbers of the same denomination? Give an example? When of different denominations? Give an example?

§ **54**. The following Tables show the different kinds of denominate numbers in general use, and also their relative values.

## ENGLISH MONEY.

The denominations of English Money, are guineas, pounds, shillings, pence, and farthings.

### TABLE.

| | | | |
|---|---|---|---|
| 4 farthings marked *far.* make | 1 penny marked | *d.* |
| 12 pence - - - | 1 shilling - | *s.* |
| 20 shillings - - - | 1 pound - | £. |
| 21 shillings - - - | 1 guinea - | |

| £ | | *s* | | *d* | | *far.* |
|---|---|---|---|---|---|---|
| 1 | = | 20 | = | 240 | = | 960 |
| | | 1 | = | 12 | = | 48 |
| | | | | 1 | = | 4 |

NOTE.—Farthings are generally expressed in fractions of a penny. Thus, for 1 farthing we write $\frac{1}{4}d$, for 2 farthings $\frac{1}{2}d$, and for 3 farthings, $\frac{3}{4}d$.

Q. What are the denominations of English Money? Repeat the table. How are farthings generally expressed?

## REDUCTION OF DENOMINATE NUMBERS.

§ **55.** Reduction is changing the denomination of a number without altering its value.

For example, 42 dollars and 35 cents are expressed in different denominations.

But 42 dollars are equal to 4200 cents,
Add 35 cents,
the sum 4235 cents is equal to 42 dollars and 35 cents. Here we have brought the numbers to the same denomination without altering their value.

Again, if we have 24 shillings, we can reduce them to pounds and shillings: for, since 20 shillings make 1 pound, 24 shillings are equal to £1 4s. Here we have again changed the denomination without altering the value.

We may take, as another example, 3 yards and reduce it to inches. Now, since 3 feet make a yard, and 12 inches a foot, we have

$3\times3=9$ feet; and $9\times12=108$ inches.

If, on the contrary, it were required to bring inches into yards, we should first divide by 12, to bring them into feet, and then by 3 to bring the feet into yards. Thus,

108 inches $\div12=9$ feet; and 9 feet $\div3=3$ yards.

We therefore see, that reduction of denominate numbers generally, like that of Federal Money, is divided into two parts.

1st. To reduce a number from a higher denomination to a lower.

2d. To reduce a number from a lower denomination to a higher.

Q. What is reduction? How many pounds and shillings in 24 shillings? How many feet in a yard? How many inches in a foot? How many feet in three yards? How many inches in 3 yards? How many feet in 72 inches? How many yards? Into how many parts may reduction of denominate numbers be divided? Name them.

### CASE I.

§ **56.** To reduce denominate numbers from a higher denomination to a lower.

### RULE.

I. *Consider how many units of the next lower denomination make one unit of the higher.*

II. *Multiply the higher denomination by that number, and add to the product the number belonging to the lower: we shall then have the equivalent number in the next lower denomination.*

III. *Proceed in a similar way through all the denominations to the last; the last sum will be the required number.*

Q. How do you reduce numbers from a higher to a lower denomination? Repeat the rule.

### EXAMPLES.

1. Reduce 9 yards and 6 feet to inches?

We first bring the yards to feet, and then add the 6 feet, after which we reduce the whole to inches.

OPERATION.

```
  9
  3
 --
 27
  6  feet to be added.
 --
 33
 12
---
396  inches.
```

2. Reduce £27 6*s* 8*d* to the denomination of pence.

We first bring the pounds to shillings and then add the 6*s*; we then bring the shillings to pence and add in the 8*d*, giving for the answer, 6560 pence.

OPERATION.

```
£27  6s  8d
 20
----
 540
   6s
----
 546s
  12
-----
6552
   8d
-----
6560d
```

In reducing, we often add the next lower denomination mentally without setting it down. Thus, when we multiply by 20, we add the 6*s*, without writing it down, making in the product 6 in the units place: and when we multiply by 12 we say, 12 times 6 are 72 and 8*d* to be added make 80.

OPERATION.

| |
|---|
| £27 6*s* 8*d* |
| 20 |
| 546*s* |
| 12 |
| 6560 |

3. In £1465 14*s* 5*d* how many farthings? *Ans.* 1407092.
4. In £45 12*s* 10*d*, how many pence? *Ans.* 10954.
5. In 87 guineas, how many farthings? *Ans.* 87696.
6. In £145 16*s* 11*d*, how many pence? *Ans.* 35003.

### CASE II.

§ 57. To reduce denominate numbers from a lower denomination to a higher.

### RULE.

I. *Consider how many units of the given denomination make one unit of the next higher; and take this number for a divisor: divide the given number by it and set down the remainder, if there be any.*

II. *Divide the quotient thus obtained by the number of units in the next higher denomination, and set down the remainder.*

III. *Proceed in the same way through all the denominations to the highest; the last quotient with the several remainders annexed, will give the answer sought, and if there be no remainders, the last quotient will be the answer.*

Q. In reducing from a lower denomination to a higher what do you first do? What next? and what next?

### EXAMPLES.

1. Reduce 3138 farthings to the denomination of pounds.

In this example we first divide by 4, the number of farthings in a penny; the quotient is 784 pence, and 2 farthings over. The 784 pence are then divided by 12, the number of pence in a shilling. The quotient is 65 shillings and four pence over.

OPERATION.

4)3138
12)784 . 2 *far.* rem.
2|0)6|5 . . 4*d.* rem.
3 . . 5*s.* rem.
*Ans.* £3 5*s* 4*d* 2 *far.*

The 65 shillings are then divided by 20, the number of shillings in a pound, the quotient is £3 and a remainder of 5 shillings. Hence, £3 5*s* 4*d* 2 *far.* is the value of 3138 farthings.

NOTE.—The same rules apply to all the denominate numbers.

2. Reduce 3658 inches to yards? *Ans.* 101 *yards*, 1 *foot*, 10 *inch.*

3. In 80 guineas, how many pounds? *Ans.* £84.

4. In 1549 farthings, how many pounds shillings and pence? *Ans.* £1 12*s* 3¼*d.*

5. Reduce 1046 pence to pounds. *Ans.* £4 7*s* 2*d.*

6. Reduce 4704 pence to guineas. *Ans.* 18 *guineas* 14*s.*

7. In 6169 pence, how many £? *Ans.* £25 14*s* 1*d.*

PROOF OF REDUCTION.

§ **58**. After a number has been reduced from a higher denomination to a lower, by the first rule, let it be reduced back by the second; and after a number has been reduced from a lower denomination to a higher, by the second rule, let it be reduced back by the first rule. If the results agree the work is supposed right.

EXAMPLES.

1. Reduce £15 7*s* 6*d* to the denomination of pence.

| OPERATION. | PROOF. |
|---|---|
| 15 | 12)3690 |
| 20 | 2\|0)30\|7 . . . 6*d* Rem. |
| 307 | 15 . . . . 7*s* Rem. |
| 12 | |
| 3690 | *Ans.* £15 7*s* 6*d.* |

2. In £31 8*s* 9*d* 3 *far.*: how many farthings? Also the proof.

3. In £87 14*s* 8½*d*: how many farthings? Also the proof.

4. In £407 19*s* 11¾*d*: how many farthings? Also the proof.

TROY WEIGHT.

§ **59**. Gold, silver, jewels, and liquors, are weighed by this weight. Its denominations are pounds, ounces, pennyweights, and grains.

TABLE.

| | | | |
|---|---|---|---|
| 24 grains, *gr.* | make | 1 pennyweight, marked | *pwt.* |
| 20 pennyweights | - | 1 ounce | *oz.* |
| 12 ounces | - | 1 pound | *lb.* |

| *lb.* | *oz.* | *pwt.* | *gr.* |
|---|---|---|---|
| 1 = | 12 = | 240 = | 5760 |
| | 1 = | 20 = | 480 |
| | | 1 = | 24 |

*Q.* What things are weighed by Troy weight? What are its denominations? Repeat the Table?

EXAMPLES.

1. Reduce 16*lb.* 11*oz.* 15*pwt.* to pennyweights.

In this example, we first multiply by the number of ounces in a pound, and then add the ounces; we then multiply by 20 and add the pennyweights.

OPERATION.

```
  16 lb.
  12 oz.
 192
  11 oz. added.
 203
  20 pwt. in an oz.
4060
  15 pwt. added.
4075 pennyweights.
```

2. In 25*lb.* 9*oz.* 0*pwt.* 20*gr.*: how many grains?

*Ans.* 148340.

3. Reduce 6490*gr.* to pounds.

We first divide by the number of grains in a *pwt.*; then by the *pwts.* in an *oz.*; then by the ounces in a *lb.*

OPERATION.

```
24) 6490
  2|0)27|0  10gr. remainder.
   12)13 . . 10pwt. remainder.
        1 . . 1oz. remainder.
```

*Ans.* 1*lb.* 1*oz.* 10*pwt.* 10*gr.*

4. In 678618 grains, how many pounds?

*Ans.* 117*lb.* 9*oz.* 15*pwt.* 18*gr.*

5. Reduce 8794*pwt.* to pounds.

*Ans.* 36*lb.* 7*oz.* 14*pwt.*

## APOTHECARIES' WEIGHT.

§ 60. This weight is used by apothecaries and physicians in mixing their medicines. Its denominations are pounds, ounces, drams, scruples, and grains. The pound and ounce are the same as the pound and ounce in the Troy weight; the difference between the two weights consists in the different divisions and subdivisions of the ounce.

### TABLE.

| | | | | |
|---|---|---|---|---|
| 20 grains, *gr.* | make | 1 scruple, | marked | ℈. |
| 3 scruples | - - | 1 dram, | - - - | ʒ. |
| 8 drams | - - - | 1 ounce, | - - - | ℥. |
| 12 ounces | - - - | 1 pound, | - - | ℔. |

| ℔ | ℥ | ʒ | ℈ | *gr.* |
|---|---|---|---|---|
| 1= | 12= | 96= | 288= | 5760 |
| | 1= | 8= | 24= | 480 |
| | | 1= | 3= | 60 |
| | | | 1= | 20 |

Q. What is the use of the Apothecaries' weight? What are its denominations? Of what value are the pound and the ounce? Repeat the Table.

### EXAMPLES.

1. Reduce 9℔ 8℥ 6ʒ 2℈ 12*gr.*, to grains.

We first multiply by the number of ounces in a *lb.*, and at the same time add in the ounces. We next multiply by the number of drams in an ounce and add in the drams: we then multiply by the number of scruples in a dram and add in the scruples; and lastly, we multiply by 20 and add in the grains.

OPERATION.

```
     9
    12
   116 ounces.
     8
   934 drams.
     3
  2804 scruples.
    20
Ans. 56092 grains.
```

2 Reduce 27℔ 9℥ 6ʒ 1℈ to scruples. *Ans.* 8011 *scruples.*

3. Reduce 94℔ 11℥ 1ʒ to drams. *Ans.* 9113 *drams.*

4. In 56092 grains, how many pounds?

We first divide by 20, the number of grains in a scruple: then by 3 the number of scruples in a dram: then by 8 the number of drams in an ounce; and lastly by 12, the number of ounces in a pound.

OPERATION.

```
2|0)5609|2
   3)2804 . . 12gr.
    8) 934 . . 2℈.
   12) 116 . . 6ʒ.
         9 ℔ 8℥.
```

*Ans.* 9℔ 8℥ 6ʒ 2℈ 12*gr.*

5. Reduce 24033 grains to pounds.

*Ans.* 4℔ 2℥ 0ʒ 1℈ 13*gr.*

6. Reduce 32044 scruples to pounds.

*Ans.* 111℔ 3℥ 1ʒ 1℈.

## AVOIRDUPOIS WEIGHT.

§ **61**. By this weight are weighed all coarse articles, such as hay, grain, chandlers' wares, and all the metals, excepting gold and silver.

Its denominations are tons, hundreds, quarters, pounds, and drams.

In this weight the words *gross* and *nett* are used. Gross is the weight of the goods, with the boxes, casks, or bags, in which they are contained. Nett is the weight of the goods only; or what remains after deducting from the gross weight, the weight of the boxes, casks, or bags.

A hundred weight is 112 pounds, as appears from the Table. But at the present time, the merchants in our principal cities, buy and sell by the 100 pounds.

### TABLE.

| | | | | |
|---|---|---|---|---|
| 16 drams *dr.* | make | 1 ounce, | marked | *oz.* |
| 16 ounces | - - | 1 pound, | - - - | *lb.* |
| 28 pounds | - - | 1 quarter, | - - - | *qr.* |
| 4 quarters | - - | 1 hundred weight, | | *cwt.* |
| 20 hundred weight | | 1 ton, | - - - - | T. |

| T. | *cwt.* | *qr.* | *lb.* | *oz.* | *dr.* |
|---|---|---|---|---|---|
| 1= | 20= | 80= | 2240= | 35840= | 573440 |
| | 1= | 4= | 112= | 1792= | 28672 |
| | | 1= | 28= | 448= | 7168 |
| | | | 1= | 16= | 256 |
| | | | | 1= | 16 |

Q. What articles are weighed by this weight? What are its denominations? What does *gross* mean? What does *net* mean? What is a hundred weight? How do they buy and sell in the principal cities? Repeat the Table.

EXAMPLES.

1. Reduce 5T. 8*cwt.* 3*qr.* 24*lb.* 13*oz.* 14*dr.* to drams.

We first multiply by 20 and add in the 8 hundred: we next multiply by 4 and add in the 3*qr.*; next by 28 and then add in the 24*lb.*; next by 16 and then add in the 13*oz*; and finally by 16 and add in the 14*dr.*

OPERATION

```
      5
     20
    ---
    108
      4
    ---
    435
     28
  -----
  12204
     16
 ------
 195277
     16
-------
3124446 drams.
```

2. Reduce 27T. 17*cwt.* 29*qr.* 21*lb.* to ounces. *Ans.* 1011472*oz.*

3. Reduce 94T. 19*cwt.* 1*qr.* to quarters. *Ans.* 7597*qr.*

4. Reduce 3124446 drams to tuns.

```
4×4=16 { 4)3124446
       {   4)781111 . . . 2
4×4=16 {   4)195277 . . . 3×4+2=14dr.
       {    4)48819 . . . 1
4×7=28 {    4)12204 . . . 3×4+1=13oz.
       {     7)3051
              4)435 . . . 6×4=24lb.
           2|0)10|8 . . . 3qr.
                 5  . . . 8cwt.
```

*Ans.* 5T. 8*cwt.* 3*qr.* 24*lb.* 13*oz.* 14*dr.*

5 Reduce 108910592 drams to tons. *Ans.* 189T. 18*cwt.* 2*qr.*

6. Reduce 2998128 ounces to tons. *Ans.* 83T. 13*cwt.* 0*qr.* 7*lb.*

## LONG MEASURE.

§ 62. Long measure is used when length only is considered. Its denominations are degrees, leagues, miles, furlongs, rods, yards, feet, inches, and barley-corns.

### TABLE.

| | | | |
|---|---|---|---|
| 3 barley-corns, *bar.* make | 1 inch, | marked | *in.* |
| 12 inches - - - | 1 foot, | - - | *ft.* |
| 3 feet - - - | 1 yard | - - | *yd.* |
| $5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet - | 1 rod, perch, or pole, | | *rd.* |
| 40 rods - - - | 1 furlong, | - - | *fur.* |
| 8 furlongs or 320 rods | 1 mile, | - - | *mi.* |
| 3 miles - - - | 1 league, | - - | L. |
| 60 geographical or $69\frac{1}{2}$ statute miles | 1 degree | - | *deg.* or ° |
| 360 degrees - - | a great circle, or circumference of the earth. | | |

| *mi.* | *fur.* | *rd.* | *yd.* | *ft.* | *in.* |
|---|---|---|---|---|---|
| 1= | 8= | 320= | 1760= | 5280= | 63360 |
| | 1= | 40= | 220= | 660= | 7920 |
| | | 1= | $5\frac{1}{2}$= | $16\frac{1}{2}$= | 198 |
| | | | 1= | 3= | 36 |
| | | | | 1= | 12 |

NOTE.—A fathom is six feet, and is generally used to measure the depth of water.

A hand is 4 inches, and is used to measure the height of horses.

Q. When is Long Measure used? What are its denominations? Repeat the table. What is a fathom? What is a hand?

### EXAMPLES.

1. In 675 *ft.* 10*in.* 2*bar.*; how many barley-corns?

We first reduce the feet to inches and then add in the 10 inches: we next reduce the inches to barley-corns and add in the 2 barleycorns.

OPERATION.

```
     675
      12
    8110
       3
Ans. 24332 barley-corns.
```

2. In 59*mi.* 7*fur.* 38*rd.*; how many rods?

*Ans.* 19198*rd.*

3. In 194656 *bar.*; how many feet?

We first divide by the number of barleycorns in an inch, and then by the number of inches in a foot.

OPERATION.

```
 3)194656
12)64885 . . . 1bar.
    5407 . . . 1in.
```

*Ans.* 5407*ft.* 1*in.* 1*bar.*

4. In 115188 rods, how many miles?

*Ans.* 359*mi.* 7*fur.* 28*rd.*

## CLOTH MEASURE.

§ **63**. Cloth measure is used for measuring all kinds of cloth. Its denominations are Ells French, Ells English, Ells Flemish, yards, quarters, nails, and inches.

### TABLE.

| | | | | | |
|---|---|---|---|---|---|
| 2¼ inches | *in.* | make | 1 nail | marked | *na.* |
| 4 nails | - | - | - 1 quarter of a yard | | *qr.* |
| 4 quarters | - | - | - 1 yard | - - | *yd.* |
| 3 quarters | - | - | - 1 Ell Flemish | - | *E. Fl.* |
| 5 quarters | - | - | - 1 Ell English | - | *E. E.* |
| 6 quarters | - | - | - 1 Ell French | - | *E. Fr.* |

Q. For what is Cloth Measure used? What are its denominations? Repeat the Table.

### EXAMPLES.

1. In 35*yd.* 3*qr.* 3*na.*, how many nails?

We first reduce the yards to quarters and add in the 3*qr.*; we next reduce the quarters to nails and add in the 3 nails.

OPERATION.

```
      35
       4
     143
       4
Ans. 575 na.
```

2. Reduce 49 Ells English to nails. *Ans.* 980*na.*

3. Reduce 51 Ells Flemish, 2*qr.* 3*na.* to nails.

*Ans.* 623*na.*

4. In 3278 nails, how many yards?

We first divide by 4 which brings the number to quarters, and then again by 4, which brings it to yards.

OPERATION.

4)3278
4)819 . . 2*na.*
204 . . 3*qr.*

*Ans:* 204*yd.* 3*qr.* 2*na.*

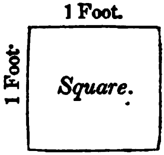
5. Reduce 340 nails to Ells Flemish. *Ans.* 28 *E. Fl.* 1*qr.*

6. In 67 quarters, how many yards? *Ans.* 16*yd.* 3*qr.*

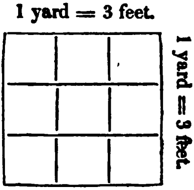
### LAND OR SQUARE MEASURE.

§ **64**. Land or square measure is used in measuring land, or any thing in which length and breadth are both considered.

A square is the space included between four equal lines, drawn perpendicular to each other. Each line is called a side of the square. If each side be one foot, the figure is called a *square foot.*




If the sides of the square be each one yard, the square is called a *square yard*. In the large square there are nine small squares, the sides of which are each one foot. Therefore, the square yard contains 9 square feet.




The number of small squares that is contained in any large square, is always equal to the product of two of the sides of the large square. As in the figure, $3\times3=9$ square feet. The number of square inches contained in a square foot is equal to $12\times12=144$.

TABLE.

| | | | |
|---|---|---|---|
| 144 square inches, *sq. in.* | make | 1 square foot, | *Sq. ft.* |
| 9 square feet | - - | 1 square yard, | *Sq. yd.* |
| 30¼ square yards | - - | 1 square pole, | *P.* |
| 40 square poles | - - | 1 rood, - | *R.* |
| 4 roods | - - - | 1 acre, - | *A.* |
| 640 acres | - - - | 1 square mile, | *M.* |

| *A.* | *R.* | *P.* | *Sq. yd.* | *Sq. ft.* | *Sq. in.* |
|---|---|---|---|---|---|
| 1= | 4= | 160= | 4840= | 43560= | 6272640 |
| | 1= | 40= | 1210= | 10890= | 1568160 |
| | | 1= | $30\frac{1}{4}$= | $272\frac{1}{4}$= | 39204 |
| | | | 1= | 9= | 1296 |
| | | | | 1= | 144 |

The Surveyor's or Gunter's chain is generally used in surveying land. It is 4 poles or 66 feet in length, and is divided into 100 links.

TABLE.

| | | | |
|---|---|---|---|
| $7\frac{92}{100}$ inches | make | 1 link, marked | *l.* |
| 4 rods or 66 *ft.* | - - - | 1 chain, - - - - | *c.* |
| 80 chains | - - - - | 1 mile, - - - - | *mi.* |
| 1 square chain | - - - | 16 square poles, | *P.* |
| 10 square chains | - - | 1 acre, - - - - | *A.* |

Land is generally estimated in square miles, acres, roods, and square poles or perches.

Q. For what is square measure used? What is a square? If each side be one foot, what is it called? If each side be a yard, what is it called? How many square feet does the square yard contain? How is the number of small squares contained in a large square found? Repeat the Table. What chain is used in surveying land? How long is it? How is it divided? Repeat the Table. How is land generally estimated?

EXAMPLES.

1. In 32M. 25A. 3R., how many square poles?

We first bring the square miles to acres by multiplying by 640, and then add in the 25 acres. We next reduce to roods and add in the 3 roods: we then reduce to poles.

OPERATION.

```
        32
       640
     -----
     20505
         4
     -----
     82023 roods,
        40
   -------
Ans. 3280920 P.
```

2. In 19A. 2R. 37P., how many square poles?
*Ans.* 3157 *P.*

3. In 175 square chains, how many square rods?
*Ans.* 2800*P*.

4. In 37456 square inches, how many square feet?

```
              ⎧ 12)37456
12×12=144     ⎨  12)3231 . . 4
              ⎩      260 . . 1
             1×12+4=16.
```

*Ans.* 260 *Sq. ft.* 16 *Sq. in.*

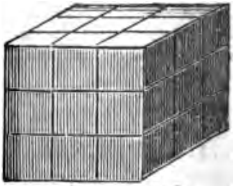
5. In 14972 square rods, how many acres?
*Ans.* 93*A.* 2*R.* 12*P.*

6. In 3674139*P.* how many square miles?,
*Ans.* 35*M.* 563*A.* 1*R.* 19*P.*

## SOLID OR CUBIC MEASURE.

§ **65.** Solid or cubic measure is used in measuring stone, timber, earth, and such other things as have three dimensions of length, breadth, and thickness. Its denominations are tons, cords, yards, feet, and inches.

A cube is a body, or solid, having six equal faces, which are squares. If the sides of the cube be each one foot long, the solid is called a cubic or solid foot. But when the sides of the cube are one yard, as in the figure, the cube is called a cubic or solid yard.



3 feet = 1 yard.

3 feet = 1 yard.

The base of the cube, which is the face on which it stands, contains $3 \times 3 = 9$ square feet. Therefore 9 cubes, of one foot each, can be placed on the base. If the solid were one foot high it would contain 9 cubic feet; if it were 2 feet thick it would contain two tiers of cubes, or 18 cubic feet; and if it were 3 feet high, it would contain three tiers, or 27 cubic feet. Hence, *the content of a solid is equal to the product of its length, breadth, and thickness.* Therefore, 1 cubic foot contains $12 \times 12 \times 12 = 1728$ cubic inches.

TABLE.

| | | | |
|---|---|---|---|
| 1728 solid inches, *S. in.* | make | 1 solid foot, | *S. ft.* |
| 27 solid feet | - - - - - | 1 solid yard | *S. yd.* |
| 40 feet of round, or 50 feet of hewn timber, | | 1 ton - - - | Ton. |
| 128 solid feet $=8\times4\times4$, that is, a pile 8 feet in length, 4 feet in width, and 4 feet in height. | | make 1 Cord of wood. - - | C. |

NOTE.—A cord foot, is one foot in length of the pile which makes a cord. It contains sixteen solid feet.

Q. For what is solid or cubic measure used? What are its denominations? What is a cube? What is a cubic or solid foot? What is a cubic yard? How many cubic feet in a cubic yard? What is the content of a solid equal to? Repeat the Table. What is a cord of wood? How many solid feet does it contain?

EXAMPLES.

1. Reduce 14 Tons of round timber to solid inches.
*Ans.* 967680 *solid inches.*
2. In 55 cords of wood, how many solid feet?
*Ans.* 7040.
3. In 25 cords of wood, how many cord feet?
*Ans.* 200 cord *ft.*
4. Reduce 3058560 cubic inches to tons of round timber.

We first divide by 1728 the number of solid inches in a solid foot, and next by 40 the number of solid feet in a ton.

OPERATION.

```
1728)3058560(1770
     1728
     -----
     13305
     12096
     ------
      12096        4|0)177|0
      12096           -------
      -----            44 . . 10
      00000
```

*Ans.* 44 tons 10*ft.*

5. Reduce 28160 solid feet to cords. *Ans.* 220 *cords.*
6. Reduce 174964 cord feet to cords.
*Ans.* 21870 *cords,* 4 *cord feet.*

## WINE MEASURE.

§ 66. Wine measure is used in measuring all liquors excepting beer and ale. Its denominations, are tuns, pipes, hogsheads, barrels, gallons, quarts, pints, and gills.

### TABLE.

| | | | |
|---|---|---|---|
| 4 gills, | *gi.* make | 1 pint, | marked *pt.* |
| 2 pints | - - - - - - | 1 quart, | - - - *qt.* |
| 4 quarts | - - - - - - | 1 gallon, | - - - *gal.* |
| 31½ gallons | - - - - - | 1 barrel, | - - *bar.* |
| 63 gallons | - - - - - | 1 hogshead, | - *hhd.* |
| 2 hogsheads | - - - - | 1 pipe, | - - - - *pi.* |
| 2 pipes or 4 hogsheads | | 1 tun, | - - - *tun.* |

| *tun.* | *pi.* | *hhd.* | *bar.* | *gal.* | *qt.* | *pt.* | *gi.* |
|---|---|---|---|---|---|---|---|
| 1 = | 2 = | 4 = | 8 = | 252 = | 1008 = | 2016 = | 8064 |
| | 1 = | 2 = | 4 = | 126 = | 504 = | 1008 = | 4032 |
| | | 1 = | 2 = | 63 = | 252 = | 504 = | 2016 |
| | | | 1 = | 31½ = | 126 = | 252 = | 1008 |
| | | | | 1 = | 4 = | 8 = | 32 |
| | | | | | 1 = | 2 = | 8 |
| | | | | | | 1 = | 4 |

NOTE.—A gallon, wine measure, contains 231 cubic inches.

Q. What is measured by wine measure? What are its denominations? Repeat the Table. What is the content of the wine gallon?

### EXAMPLES.

1. In 5 tuns 1 hogshead of wine, how many gallons?

We first multiply by 4 the number of hogsheads in a tun and add in the 1 hogshead, after which we reduce to gallons.

OPERATION.

```
      5
      4
     --
     21
     63
     --
     63
    126
   ----
Ans. 1323 gal.
```

2. Reduce 12 pipes 1 hogshead and 1 quart of wine, to pints. *Ans.* 12602*pt.*

3. In 1 tun of cider, how many gills? *Ans.* 8064.

4. In 10584 quarts of wine, how many tuns?

```
            4)10584
           ____________
          ( 7)2646
63=7×9 {   ____________
          (  9)378
             ____________
              4)42
             ____________
     Ans. 10tuns. 2hhd.
```

5. Reduce 201632 gills to tuns. *Ans.* 25*tuns.* 1*gal.*
6. Reduce 16128 gills of cider to tuns. *Ans.* 2*tuns.*

### ALE OR BEER MEASURE.

§ 67. Ale or beer measure is used in measuring ale, beer, and milk. Its denominations are hogsheads, barrels, gallons, quarts, and pints.

TABLE.

| | | | | |
|---|---|---|---|---|
| 2 pints | *pt.* make | 1 quart, | marked | *qt.* |
| 4 quarts | - - - - | 1 gallon, | - - - | *gal.* |
| 36 gallons | - - - - | 1 barrel, | - - - | *bar.* |
| 54 gallons | - - - - | 1 hogshead, | - - | *hhd.* |

| *hhd.* | *bar.* | *gal.* | *qt.* | *pt.* |
|---|---|---|---|---|
| 1 = | $1\frac{1}{2}$ = | 54 = | 216 = | 432 |
| | 1 = | 36 = | 144 = | 288 |
| | | 1 = | 4 = | 8 |
| | | | 1 = | 2 |

NOTE.—A gallon, beer measure, contains 282 cubic inches.

Q. For what is ale or beer measure used? What are its denominations? Repeat the Table. How many cubic inches in a gallon, beer measure?

EXAMPLES.

1. Reduce 47*bar.* 16*gal.* 4*qt.* to pints. *Ans.* 13672*pt.*
2. In 27*hhd.* of beer, how many pints? *Ans.* 11664.
3. In 55832 pints of beer, how many hogsheads?
*Ans.* 129*hhd.* 13*gal.*
4. In 64972 quarts of beer, how many barrels?
*Ans.* 451*bar.* 7*g.*

## DRY MEASURE.

§ **68**. Dry measure is used in measuring all dry articles, such as grain, fruits, roots, salt, coal, &c. Its denominations are chaldrons, bushels, pecks, quarts, and pints.

### TABLE.

| | | | | |
|---|---|---|---|---|
| 2 pints *pt.* | make | 1 quart, | marked | *qt.* |
| 8 quarts | - | 1 peck, | - | *pk.* |
| 4 pecks | - | 1 bushel, | - | *bu.* |
| 36 bushels | - | 1 chaldron, | - | *ch.* |

| *ch.* | *bu.* | *pk.* | *qt.* | *pt.* |
|---|---|---|---|---|
| 1= | 36= | 144= | 1152= | 2304 |
| | 1= | 4= | 32= | 64 |
| | | 1= | 8= | 16 |
| | | | 1= | 2 |

NOTE.—A gallon Dry measure contains $268\frac{4}{5}$ cubic inches. A Winchester bushel is $18\frac{1}{2}$ inches in diameter, 8 inches deep, and contains $2150\frac{2}{5}$ cubic inches.

Q. What is the use of Dry measure? What are its denominations? Repeat the Table. What is the content of a gallon? How large is a Winchester bushel?

### EXAMPLES.

1. In 372 bushels, how many pints? *Ans.* 23808.
2. In 5 chaldrons 31 bushels, how many pecks? *Ans.* 844.
3. In 17408 pints, how many bushels? *Ans.* 272.
4. In 4220 pints, how many chaldrons? *Ans.* 1*ch.* 29*bu.* 3*pk.* 6*qt.*

## TIME.

§ **69**. The denominations of time are years, months, weeks, days, hours, minutes, and seconds.

| | | | | |
|---|---|---|---|---|
| 60 seconds *sec.* | make | 1 minute, | marked | *m.* |
| 60 minutes | - - - - - | 1 hour, | - - - - - | *hr.* |
| 24 hours | - - - - - - | 1 day, | - - - - - | *da.* |
| 7 days | - - - - - - | 1 week, | - - - - - | *wk.* |
| 4 weeks | - - - - - | 1 month, | - - - - - | *mo.* |
| 13 mo. 1 day and 6 hrs. or 365 days, 6 hours. | | 1 common or Julian year, | - - | *yr.* |

| yr. | wk. | da. | hr. | m. | sec. |
|---|---|---|---|---|---|
| 1= | 52= | $365\frac{1}{4}$= | 8766= | 525960= | 31557600 |
| | 1= | 7 = | 168= | 10080= | 604800 |
| | | 1 = | 24= | 1440= | 86400 |
| | | | 1= | 60= | 3600 |
| | | | | 1= | 60 |

The whole days only are reckoned. The odd six hours, by accumulating for 4 years, make one day, so that every fourth year contains 366 days. This is called the Bissextile, or Leap year.

When the Leap year occurs, the number of years up to that time is exactly divisible by 4. Thus 1800, 1804, 1808, 1812, 1816, 1820, 1824, 1828, 1832, and 1836 were all leap years.

The year is also divided into 12 callendar months, which contain an unequal number of days.

| | | *Names.* | *No. of Days.* |
|---|---|---|---|
| 1 | month | January, | 31 |
| 2 | - - - | February, | 28 |
| 3 | - - - | March, | 31 |
| 4 | - - - | April, | 30 |
| 5 | - - - | May, | 31 |
| 6 | - - - | June, | 30 |
| 7 | - - - | July, | 31 |
| 8 | - - - | August, | 31 |
| 9 | - - - | September, | 30 |
| 10 | - - - | October, | 31 |
| 11 | - - - | November, | 30 |
| 12 | - - - | December, | 31 |
| | | Total | 365 |

The additional day, when it occurs, is added to the month of February, so that this month has 29 days in the Leap year.

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February, twenty-eight alone.

Q. What are the denominations of Time? How long is a year? How many days in a common year? How many days in a leap year? How many calendar months in a year? Name them and the number of days in each. How many days has February in the leap year? How do you remember which of the months have 30 days, and which 31?

EXAMPLES.

1. How many seconds in a year of 365*da.* 6*hr.*?

We first reduce the days to hours and add in the 6 hours. We then multiply by 60 which brings the whole to minutes, after which we again multiply by 60 which reduces the number to seconds.

OPERATION.

```
     365da. 6hr.
      24
    1466
    730
    8766  hours.
      60
  525960  minutes.
      60
Ans. 31557600 seconds.
```

2. In 12 years of 365*da.* 23*hr.* 57*m.* 39*sec.* each, how many seconds? *Ans.* 379467108*sec.*

3. In 49 weeks, how many minutes? *Ans.* 493920.

4. In 126230400 seconds, how many years of 365 days?

We first divide by 60 which brings the number into minutes. We then divide again by 60 which brings it into hours, then by 24 which brings it into days; and lastly, by 365 which gives the quotient in years.

OPERATION.

```
             6|0)12623040|0
              6|0)210384|0
4×6=24 {      4)35064
               6)8766
           365)1461(4
               1460
                  1
```

*Ans.* 4 *years* and 1 *day.*

5. In 756952018 seconds, how many years of 365 days each? *Ans.* 24*yr.* 1*da.* 26*m.* 58*sec.*

6. In 5927040 minutes, how many weeks? *Ans.* 588.

## CIRCULAR MEASURE OR MOTION.

§ **70**. Circular measure is used in estimating latitude and longitude, and also in measuring the motions of the heavenly bodies. Every circle is supposed to be divided into 360 equal parts, called degrees.

### TABLE.

| | | | |
|---|---|---|---|
| 60 seconds ″ | make | 1 minute, | marked ′. |
| 60 minutes | - - | 1 degree, | - - °. |
| 30 degrees | - - | 1 sign, | - - - s. |
| 12 signs or 360° | - | 1 circle, | - - - c. |

| c. | s. | °. | ′. | ″. |
|---|---|---|---|---|
| 1 = | 12 = | 360 = | 21600 = | 1296000 |
| | 1 = | 30 = | 1800 = | 108000 |
| | | 1 = | 60 = | 3600 |
| | | | 1 = | 60 |

Q. For what is circular measure used? How is every circle supposed to be divided? Repeat the Table.

#### EXAMPLES.

1. Reduce 5s 29° 25′ to minutes. *Ans.* 10765′.
2. In 2 circles, how many seconds? *Ans.* 2592000″.
3. In 32295 minutes how many circles? *Ans.* 1c. 5s. 28°. 15′.
4. In 27894 seconds, how many degrees? *Ans.* 7°. 44′. 54″.

### TABLE OF PARTICULARS.

| | | |
|---|---|---|
| 12 things | make | 1 dozen. |
| 12 dozen | - - - - - - | 1 gross. |
| 12 gross, or 144 dozen | - | 1 great gross |
| | ALSO, | |
| 20 things | make | 1 score. |
| 112 pounds | - - - - - - | 1 quintal of fish. |
| 24 sheets of paper | - - | 1 quire. |
| 20 quires | - - - - - - | 1 ream. |

Q. How many things make a dozen? How many dozen a gross? How many gross a great gross? How many things make a score? How many pounds a quintal of fish? How many sheets a quire of paper? How many quires a ream?

### BOOKS.

A sheet folded in two leaves is called a folio.

| | | |
|---|---|---|
| " folded in four leaves | - | a quarto, or 4to. |
| " folded in eight leaves | - | an octavo, or 8vo. |
| " folded in twelve leaves | - | a duodecimo or 12mo. |
| " folded in eighteen leaves | | an 18mo. |

**Q. When a sheet is folded in two leaves, what is it called? When folded in four leaves? When folded in eight leaves? When folded in twelve? When folded in eighteen?**

### APPLICATIONS IN REDUCTION.

1. In 6169 pence, how many pounds? *Ans.* £25 14*s* 1*d*.

2. In 59*lb*. 13*pwt*. 5*gr*., how many grains? *Ans.* 340157.

3. In £85 8*s*, how many guineas? *Ans.* 81 *guineas* 7*s*.

4. How many strokes does a regular clock strike in a year of 365 days, striking once at one, twice at two, &c., for the 24 hours in the day? *Ans.* 56940.

5. In 757 boxes of sugar containing 28*lb*. each, how many tons? *Ans.* 9*tons*, 9*cwt*. 1*qr*.

6. How many cords are there in a pile of wood that is 36 feet long, 6 feet high, and 4 feet wide? *Ans.* 6 *cords*, and 6 *cord feet*.

7. A man has a journey to perform of 288 miles; supposing him to travel 12 hours each day for 6 days in succession, at what rate must he travel per hour to accomplish it in that time? *Ans.* 4 *miles*.

8. How many yards of carpeting which is one yard in width, will be required to carpet a room 18 feet wide and 20 feet long? *Ans.* 40 *yards*.

9. Reduce 346 Ells Flemish to Ells English. *Ans.* 207$\frac{3}{5}$ *Ells Eng*.

10. Suppose the number of inhabitants in the United States to be 12 millions, how long would it take to count them, counting at the rate of 50 a minute? *Ans.* 166 *days* 16 *hours*.

11. A merchant wishes to bottle a cask of wine containing 63 gallons, into bottles containing one pint each: how many bottles are necessary? *Ans.* 504.

12. There is a cube, or square piece of wood, 2 feet or 24 inches each way; how many small cubes of one inch each way can be sawed from it, allowing no waste in sawing? *Ans.* 13824.

13. A merchant wishes to ship 285 bushels of flax seed, in casks containing 7 bushels 2 pecks each: what number of casks are required? *Ans.* 38.

## ADDITION OF DENOMINATE NUMBERS.

1. John buys a knife for 1*s* 8*d*, and a bunch of quills for 1*s* 2*d*: what do they cost him? *Ans.* 2*s* 10*d*.

2. James gives 4*s* 9*d* for a pair of shoes, and 2*s* and 4*d* for a pair of stockings: how much do they cost him? *Ans.* 7*s* 1*d*.

3. How many hours in 8*hr*.+6*hr*.+7*hr*.+9*hr*.? *Ans.* 30*hr*.

4. In 8*yd*.+7*yd*.+5*yd*.+6*yd*.: how many yards? *Ans.* 26*yds*.

5. How many pounds shillings and pence, in £4 8*s* 9*d*, £27 14*s* 11*d*, and £156 17*s* 10*d*. ?

OPERATION.

| £ | *s* | *d* |
|---|---|---|
| 4 | 8 | 9 |
| 27 | 14 | 11 |
| 156 | 17 | 10 |
| £189 | 1*s* | 6*d*. |

We write the denominations under each other, and draw a line beneath them. We then add up the column of pence, and find the sum to be 30. But 30 pence are equal to 2 shillings and 6 pence: we therefore write down the 6 and carry 2 to the shillings. We then find the sum of the shillings to be 41: that is, 2 pounds and 1 shilling over. Carrying the 2 to the column of pounds, we find the sum to be 189.

§ **71**. Addition of denominate numbers, like that of simple numbers, teaches how to express the value of several numbers by a single one, which is called their sum.

### RULE.

I. *Set down the numbers to be added so that all the denominations of the same kind shall stand in the same column.*

II. *Begin with the column of the lowest denomination, and add it up as in whole numbers.*

III. *Then consider how many units of this denomination make one unit of the next higher, and divide the sum by this number. Write down the remainder under the units of its kind, and carry the quotient to the next column, as in addition of simple numbers.*

IV. *Proceed in the same way for all the columns to the last, and set down the entire sum of the last column.*

The proof is the same as in the addition of simple numbers.

Q. What is Addition of Denominate Numbers? How do you set down the numbers for Addition? Where do you begin to add? What do you do with the first sum? What do you write down? What do you carry to the next column? How do you prove Addition?

### EXAMPLES.

1. What is the sum of £16 18*s* 9*d*, £14 13*s* 8*d*, and £15 17*s* 6*d*?

NOTE.—In simple numbers 10 units of any one of the columns, make one unit of the next left hand column § 8. We therefore carry one for every 10. But in denominate numbers the higher denominations are formed differently. For example, 12 pence make 1 shilling, the unit of the next higher denomination; and 20 shillings make 1 pound. In passing from pence to shillings, we must therefore carry 1 for every 12, and 1 for every 20 in passing from shillings to pounds. And in general, we must carry 1 for so many units of the lower denomination as make one unit of the next higher.

OPERATION.

| | £ | *s* | *d* |
|---|---|---|---|
| | 16 | 18 | 9 |
| | 14 | 13 | 8 |
| | 15 | 17 | 6 |
| Sum | 47 | 9 | 11 |
| | 30 | 11 | 2 |
| Proof | 47 | 9 | 11 |

Q. In simple numbers, how many units of one order make one unit of the next higher order? How do you carry in simple numbers? How do you carry in passing from pence to shillings? In passing from shillings to pounds? Generally, how do you carry?

| (2.) £ | s | d | (3.) £ | s | d | (4.) £ | s | d |
|---|---|---|---|---|---|---|---|---|
| 173 | 13 | 5 | 705 | 17 | 3½ | 104 | 18 | 9½ |
| 87 | 17 | 7¾ | 354 | 17 | 2¾ | 404 | 17 | 8¾ |
| 75 | 18 | 7½ | 175 | 17 | 3¾ | 467 | 11 | 10¼ |
| 25 | 17 | 8¼ | 87 | 19 | 7½ | 597 | 14 | 4¼ |
| 10 | 10 | 10½ | 52 | 12 | 7¾ | 22 | 18 | 5 |
| 373 | 18 | 3 | 1377 | 4 | 1¼ | | | |

TROY WEIGHT.

OPERATION.

| lb. | oz. | pwt. | gr. |
|---|---|---|---|
| 11 | 8 | 18 | 19 |
| 114 | 9 | 6 | 16 |
| 223 | 10 | 17 | 12 |
| 350 | 5 | 2 | 23 |

Adding up the grains, we find their sum to be 47; that is, 1*pwt.* and 23*gr.*: setting down 23, and carrying 1 to the pennyweights, we find their sum to be 42: that is, 2*oz.* and 2*pwt.* Carrying 2 to the ounces, we find their sum to be 29; that is, 2*lb.* and 5*oz.*: carrying 2 to the pounds and adding, we find their sum to be 350.

| | lb. | oz. | pwt. | gr. | lb. | oz. | pwt. | gr. |
|---|---|---|---|---|---|---|---|---|
| Add | 100 | 10 | 19 | 20 | 171 | 6 | 13 | 14 |
| | 432 | 6 | 0 | 5 | 391 | 11 | 9 | 12 |
| | 80 | 3 | 2 | 1 | 230 | 6 | 6 | 13 |
| | 7 | 0 | 0 | 9 | 94 | 7 | 3 | 18 |
| | 0 | 11 | 10 | 23 | 42 | 10 | 15 | 20 |
| | 0 | 0 | 8 | 9 | 31 | 0 | 0 | 21 |
| Sum | 621 | 8 | 1 | 19 | 962 | 6 | 10 | 2 |

APOTHECARIES' WEIGHT.

| ℔ | ℥ | ʒ | ℈ | gr. | ℥ | ʒ | ℈ | gr. | ʒ | ℈ | gr. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 24 | 7 | 2 | 1 | 16 | 11 | 2 | 1 | 17 | 3 | 2 | 15 |
| 17 | 11 | 7 | 2 | 19 | 7 | 4 | 2 | 14 | 0 | 1 | 13 |
| 36 | 6 | 5 | 0 | 7 | 4 | 0 | 1 | 19 | 2 | 2 | 11 |
| 15 | 9 | 7 | 1 | 13 | 2 | 5 | 2 | 11 | 7 | 0 | 17 |
| 9 | 3 | 4 | 1 | 9 | 10 | 1 | 2 | 16 | 5 | 2 | 14 |
| 16 | 10 | 3 | 2 | 17 | 8 | 7 | 1 | 13 | 6 | 1 | 0 |

### AVOIRDUPOIS WEIGHT.

| *cwt.* | *qr.* | *lb.* | *oz.* | *dr.* |
|---|---|---|---|---|
| 14 | 1 | 25 | 14 | 9 |
| 13 | 2 | 20 | 1 | 15 |
| 9 | 3 | 6 | 7 | 3 |
| 10 | 0 | 18 | 12 | 11 |
| 7 | 2 | 27 | 3 | 2 |
| 6 | 1 | 19 | 8 | 1 |
| 4 | 3 | 0 | 15 | 5 |
| 12 | 2 | 0 | 0 | 13 |
| 79 | 2 | 6 | 15 | 11 |

| *ton.* | *cwt.* | *qr.* | *lb.* | *oz.* |
|---|---|---|---|---|
| 15 | 12 | 1 | 10 | 10 |
| 71 | 8 | 2 | 6 | 0 |
| 83 | 19 | 3 | 15 | 5 |
| 36 | 7 | 0 | 20 | 14 |
| 47 | 11 | 1 | 27 | 11 |
| 63 | 5 | 2 | 19 | 7 |
| 12 | 13 | 1 | 14 | 9 |
| 9 | 7 | 0 | 5 | 10 |
| 340 | 5 | 2 | 8 | 2 |

A merchant bought 4 barrels of potash of the following weights, viz. 1st, 3*cwt.* 1*qr.* 25*lb.* 12*oz.* 3*dr.*; 2d, 4*cwt.* 1*qr.* 21*lb.* 4*oz.*; 3d, 4*cwt.*; 4th, 3*cwt.* 3*qr.* 27*lb.* 15*oz.* 15*dr.*: What was the entire weight of the four barrels?

*Ans.* 15*cwt.* 3*qr.* 19*lb.* 0*oz.* 2*dr.*

### LONG MEASURE.

| *L.* | *mi.* | *fur.* | *rd.* |
|---|---|---|---|
| 16 | 2 | 7 | 39 |
| 327 | 1 | 2 | 20 |
| 87 | 0 | 1 | 15 |
| 1 | 1 | 1 | 1 |
| 432 | 2 | 4 | 35 |

| *yd.* | *ft.* | *in.* | *bar.* |
|---|---|---|---|
| 90 | 2 | 11 | 2 |
| 155 | 1 | 9 | 1 |
| 327 | 0 | 7 | 0 |
| 50 | 2 | 1 | 2 |
| 624 | 1 | 5 | 2 |

### CLOTH MEASURE.

| *E. Fl.* | *qr.* | *na.* |
|---|---|---|
| 126 | 4 | 4 |
| 65 | 3 | 1 |
| 72 | 1 | 3 |
| 157 | 2 | 2 |
| 424 | 0 | 2 |

| *yd.* | *qr.* | *na.* |
|---|---|---|
| 4 | 3 | 2 |
| 5 | 4 | 1 |
| 6 | 1 | 0 |
| 25 | 2 | 2 |
| 42 | 3 | 1 |

| *E. E.* | *qr.* | *na.* |
|---|---|---|
| 128 | 5 | 1 |
| 20 | 3 | 1 |
| 19 | 1 | 4 |
| 15 | 3 | 1 |
| 184 | 3 | 3 |

### LAND OR SQUARE MEASURE.

| *Sq. yd.* | *Sq. ft.* | *Sq. in.* |
|---|---|---|
| 97 | 4 | 104 |
| 22 | 3 | 27 |
| 105 | 8 | 2 |
| 37 | 7 | 127 |
| 263 | 5 | 116 |

| *M.* | *A.* | *R.* | *P.* |
|---|---|---|---|
| 1 | 700 | 3 | 37 |
| 6 | 375 | 2 | 25 |
| 7 | 450 | 1 | 31 |
| 11 | 30 | 0 | 25 |
| 27 | 277 | 0 | 38 |

There are 4 fields, the 1st, contains 12A. 2R. 38P.; the 2d, 4A. 1R. 26P.; the 3d, 85A. 0R. 19P.; and the 4th. 57A. 1R. 2P.: how many acres in the four fields?

*Ans.* 159A. 2R. 5P.

## SOLID OR CUBIC MEASURE.

| *S. yd.* | *S. ft.* | *S. in.* |
|---|---|---|
| 65 | 25 | 1129 |
| 37 | 26 | 132 |
| 50 | 1 | 1064 |
| 22 | 19 | 17 |
| 176 | 18 | 614 |

| *C.* | *S. ft.* |
|---|---|
| 16 | 127 |
| 17 | 12 |
| 18 | 119 |
| 37 | 104 |
| 90 | 106 |

| *C.* | *Cord feet.* |
|---|---|
| 87 | 9 |
| 26 | 7 |
| 16 | 6 |
| 19 | 5 |
| 151 | 3 |

## WINE MEASURE.

| *hhd.* | *gal.* | *qt.* | *pt.* |
|---|---|---|---|
| 127 | 65 | 3 | 2 |
| 12 | 60 | 2 | 3 |
| 450 | 29 | 0 | 1 |
| 21 | 0 | 2 | 3 |
| 14 | 39 | 1 | 2 |
| 627 | 7 | 1 | 1 |

| *tun.* | *pi.* | *hhd.* | *gal.* | *qt.* |
|---|---|---|---|---|
| 14 | 2 | 1 | 27 | 3 |
| 15 | 1 | 2 | 25 | 2 |
| 4 | 2 | 1 | 27 | 1 |
| 5 | 0 | 1 | 62 | 3 |
| 7 | 1 | 2 | 21 | 2 |
| 50 | 0 | 1 | 38 | 3 |

## DRY MEASURE.

| *ch.* | *bu.* | *pk.* | *qt.* | *pt.* |
|---|---|---|---|---|
| 27 | 25 | 3 | 7 | 1 |
| 59 | 21 | 2 | 6 | 3 |
| 2 | 1 | 2 | 7 | 1 |
| 5 | 9 | 1 | 8 | 2 |
| 94 | 22 | 3 | 7 | 1 |

| *ch.* | *bu.* | *pk.* | *qt.* | *pt.* |
|---|---|---|---|---|
| 141 | 36 | 3 | 7 | 2 |
| 21 | 32 | 2 | 4 | 1 |
| 85 | 9 | 1 | 0 | 3 |
| 10 | 4 | 4 | 1 | 3 |
| 259 | 12 | 0 | 0 | 1 |

## TIME.

| *yr.* | *mo.* | *wk.* | *da.* | *hr.* |
|---|---|---|---|---|
| 4 | 11 | 3 | 6 | 20 |
| 3 | 10 | 2 | 5 | 21 |
| 5 | 8 | 1 | 4 | 19 |
| 101 | 9 | 3 | 7 | 23 |
| 55 | 8 | 4 | 6 | 17 |
| 172 | 2 | 1 | 4 | 4 |

| *wk.* | *da.* | *hr.* | *m.* | *sec.* |
|---|---|---|---|---|
| 8 | 8 | 14 | 55 | 57 |
| 10 | 7 | 23 | 57 | 49 |
| 20 | 6 | 14 | 42 | 01 |
| 6 | 5 | 23 | 19 | 59 |
| 2 | 2 | 20 | 45 | 48 |
| 50 | 4 | 1 | 41 | 34 |

## SUBTRACTION OF DENOMINATE NUMBERS.

1. John has 3*s* 6*d* and gives 1*s* 4*d* for a knife: how much has he left? *Ans.* 2*s* 2*d.*

2. James has 4*s* 8*d* and gives 2*s* 3*d* for a bunch of quills: how much has he left? *Ans.* 2*s* 5*d.*

3. What is the difference between £27 16*s* 8*d* and £19 17*s* 9*d*.

OPERATIONS.

| £ | s | d | £ | s | d |
|---|---|---|---|---|---|
| 27 | 16 | 8 | 26 | 35 | 20 |
| 19 | 17 | 9 | 19 | 17 | 9 |
| 7 | 18 | 11 | 7 | 18 | 11 |

In this example we cannot take 9*d* from 8*d*; we therefore add 12*d* to the 8*d* making 20*d*, and then say, 9 from 20, 11 remains. Set down the 11, and carry 1 to 17 making 18: then say, 18 from 36 leaves 18: set it down and carry one to 19 making 20: 20 from 27 leaves 7.

Or we may set down the minuend as on the right: £26 35*s* 20*d*. Then 9 from 20 leaves 11; 17 from 35 leaves 18; and 19 from 26, 7 remains.

§ **72**. Hence; for the subtraction of denominate numbers we have the following

### RULE.

I. *Set down the lesser number under the greater, placing the same denominations directly under each other.*

II. *Begin with the lowest denomination, and if the number expressing that denomination be less than the number directly over it, make the subtraction as in simple numbers. But if it be greater, subtract it from the upper number increased by so many units as make one unit of the next higher denomination, and carry this one which has been borrowed to the next higher denomination, as in subtraction of simple numbers.*

III. *Do the same for all the denominations, and set down the several remainders, and they will form the true remainder.*

### PROOF.

Add the remainder to the subtrahend—their sum should be equal to the minuend.

**Q.** How do you write down the numbers for subtraction? Where do you begin to subtract? When the number to be subtracted is less than the one above it, what do you do? When it is greater, what do you do? How do you prove Subtraction?

EXAMPLES.

(1.)

| | A. | R. | P. |
|---|---|---|---|
| From - - | 18 | 3 | 28 |
| Take - - | 15 | 2 | 30 |
| Remainder | 3 | 0 | 38 |
| Proof - - | 18 | 3 | 28 |

(2.)

| | T. | *cwt.* | *qr.* | *lb.* |
|---|---|---|---|---|
| From | 4 | 12 | 3 | 20 |
| Take | 2 | 18 | 2 | 26 |
| Remainder | 1 | 14 | 0 | 22 |
| Proof | 4 | 12 | 3 | 20 |

(3.)

| | *lb.* | *oz.* | *pwt.* | *gr.* |
|---|---|---|---|---|
| From - - | 273 | 0 | 0 | 0 |
| Take - - | 98 | 10 | 18 | 21 |
| Remainder | 174 | 1 | 1 | 3 |

(4.)

| | *lb.* | *oz.* | *pwt.* | *gr.* |
|---|---|---|---|---|
| From | 18 | 9 | 10 | 8 |
| Take | 9 | 10 | 15 | 20 |
| Remainder | 8 | 10 | 14 | 12 |

(5.)

| | T. | *cwt.* | *qr.* | *lb.* | *oz.* |
|---|---|---|---|---|---|
| From - - | 7 | 14 | 1 | 3 | 6 |
| Take - - | 2 | 6 | 3 | 4 | 11 |
| Remainder | 5 | 7 | 1 | 26 | 11 |

(6.)

| | *cwt.* | *qr.* | *lb.* | *oz.* | *dr.* |
|---|---|---|---|---|---|
| From | 14 | 2 | 12 | 10 | 8 |
| Take | 6 | 3 | 16 | 15 | 3 |
| Remainder | 7 | 2 | 23 | 11 | 5 |

7. From 38*mo.* 2*wk.* 3*da.* 7*hr.* 10*m.*, take 10*mo.* 3*wk.* 2*da.* 10*hr.* 50*m.* *Ans.* 27*mo.* 3*wk.* 20*hr.* 20*m.*

8. From 176*yr.* 8*mo.* 3*wk.* 4*da.*, take 91*yr.* 9*mo.* 2*wk.* 6*da.* *Ans.* 84*yr.* 11*mo.* 5*da.*

9. From 6 *tuns*, take 3*hhd.* 15*gal.* 3*qt.* *Ans.* 5 *tuns.* 47*gal.* 1*qt.*

10. From £3, take 3*s.* *Ans.* £2 17*s.*

11. From 2*lb.*, take 20*gr.* Troy. *Ans.* 1*lb.* 11*oz.* 19*pwt.* 4*gr.*

12. From 8℔, take 1℔ 1℥ 2ʒ 2℈. *Ans.* 6℔ 10℥ 5ʒ 1℈.

13. From 9T., take 1T. 1*cwt.* 2*qr.* 20*lb.* 15*oz.* 14*dr.* *Ans.* 7T. 18*cwt.* 1*qr.* 7*lb.* 0*oz.* 2*dr.*

14. From 3 miles, take 3*fur.* 19*rd.* *Ans.* 2*mi.* 4*fur.* 21*rd.*

## APPLICATIONS IN ADDITION AND SUBTRACTION.

1. Sold a merchant one quarter of beef for £2 7*s* 9*d*; one cheese for 9*s* 7*d*; 20 bushels of corn for £4 10*s* 11*d*; and 40 bushels of wheat for £19 12*s* 8½*d*: how much did the whole come to? *Ans.* £27 0*s* 11½*d*.

2. Bought of a silversmith a tea pot, weighing 3*lb.* 4*oz.* 9*pwt.* 21*gr.*; one dozen of silver spoons, weighing 2*lb.* 1*oz.* 1*pwt.*; 2 dishes weighing 16*lb.* 10*oz.* 15*pwt.* 16*gr.*: how much did the whole weigh?
*Ans.* 22*lb.* 4*oz.* 6*pwt.* 13*gr.*

3. Bought one hogshead of sugar, weighing 9*cwt.* 2*qr.* 27*lb.* 14*oz.*; one barrel weighing 3*cwt.* 27*lb.*, and a second barrel weighing 2*cwt.* 3*qr.* 26*lb.* 4*oz.*: how much did the whole weigh? *Ans.* 15*cwt.* 3*qr.* 25*lb.* 2*oz.*

4. A merchant buys two hogsheads of sugar, one weighing 8*cwt.* 3*qr.* 21*lb.*; the other, 9*cwt.* 2*qr.* 6*lb.*; he sells two barrels, one weighing 3*cwt.* 1*qr.* 12*lb.* 14*oz.*; the other, 2*cwt.* 3*qr.* 15*lb.* 6*oz.*: how much remains on hand?
*Ans.* 12*cwt.* 26*lb.* 12*oz.*

5. A man sets out upon a journey and has 200 miles to travel; the first day he travels 9 leagues 2 miles 7 furlongs 30 rods; the second day 12 leagues 1 mile 1 furlong; the third day 14 leagues; the fourth day 15 leagues 2 miles 5 furlongs 35 rods: how far had he then to travel?
*Ans.* 14L. 1*mi.* 1*fur.* 15*rd.*

6. A farmer has two meadows, one containing 9*A.* 3*R.* 37*P.*, the other contains 10*A.* 2*R.* 25*P.*; also three pastures, the first containing 12*A.* 1*R.* 1*P.*, the second containing 13*A.* 3*R.*, and the third 6*A.* 1*R.* 39*P.*: by how many acres does the pasture exceed the meadow land? *Ans.* 11*A.* 3*R.* 18*P.*

7. Supposing the Declaration of Independence to have been published at precisely 12 o'clock on the 4th of July 1776, how much time elapsed to the 1st of January 1833, at 25 minutes past 3 P. M.? *Ans.* 56*yr.* 181*da.* 3*hr.* 25*m.*

8. A farmer has three granaries, one for wheat, one for rye, and one for corn: he fills them all. His wheat granary contains 657*bu.* 3*pk.* 6*qt.*; the corn granary 257*bu.* 1*pk.* 1*qt.*; the rye granary 459*bu.* 2*pk.* 7*qt.*; how

much grain had he in all, and how much more wheat than rye? *Ans.* { In all 1374*bu.* 3*pk.* 6*qt.*
Wheat more than rye 198*bu.* 7*qt.* }

9. A father was born on the 8th of December 1759, his first son on the 4th of June 1795: what was the difference of their ages? *Ans.* 35*yr.* 5*mo.* 27*da.*

10. A merchant has a bill to pay of £600. He has £250 19*s* 8*d* in cash, a good note against A for £75 10*s* 6*d* and a note against B for £37 11*s* 9*d*: how much money does he want to make the payment? *Ans.* £235 18*s* 1*d.*

11. A tailor requires 1*yd.* 3*qr.* 3*na.* of cloth for a father's coat and 1*yd.* 1*qr.* 2*na.* for each of two sons: the father buys 6 yards, does he buy too much or too little?

*Ans.* 1*yd.* 1*qr.* 1*na.* too much.

## MULTIPLICATION OF DENOMINATE NUMBERS.

1. Charles pays 6*d* for a pencil: How much will buy two pencils? How much will buy 3 pencils? 4 pencils? 5 pencils? 6 pencils?

2. James puts 1 quart and 1 pint into a measure: How much could he put in a measure of twice the size? In a measure of three times the size? 4 times the size? 5 times the size? 6 times the size?

3. What is the product of 2*s* 4*d* multiplied by 2? by 3? by 4? by 5? by 6? by 7? by 8? by 9?

4. What is the product of 1*yd.* 1*qr.* multiplied by 2? by 3? by 4? by 5? by 6? by 7? by 8? by 9?

5. Multiply £3 9*s* 10*d* by 4?

In this example we say, 4 times 10*d* are 40*d*, equal to 3*s* and 4*d*. Set down the 4*d* in the lower line. Then 4 times 9*s* are 36*s* and 3*s* to carry make 39*s*, equal to £1 and 19*s* over: set down the 19*s*. Then 4 times £3 are £12 and £1 to carry make £13.

OPERATION.

| £ | *s* | *d* |
|---|---|---|
| 3 | 9 | 10 |
| | | 4 |
| £12 | 36*s* | 40*d* |
| £13 | 19*s* | 4*d* |

We may conclude from the examples that, to multiply a denominate number by a simple one, is to repeat the denominate number as many times as there are units in the multiplier.

CASE I.

§ 73. When the simple number does not exceed 12.

RULE.

I. *Write down the denominate number and set the multiplier under the lowest denomination.*

II. *Multiply the lowest denomination by the multiplier, and see how many units of the next higher denomination are contained in the product, and set down the excess as in addition.*

III. *Multiply the next higher denomination by the multiplier and add the units to be carried from the last product; then reduce the sum to units of the next higher denomination, write down the excess and proceed in the same way for all the denominations, setting down the entire product when you come to the last.*

Q. What is required when you multiply a denominate number by a simple one? When the simple number does not exceed 12, how do you write it down? How do you begin to multiply? How do you carry?

EXAMPLES.

(1.)

| £ | s | d |
|---|---|---|
| 17 | 15 | 9 |
| | | 6 |
| 106 | 14 | 6 |

(2.)

| T. | cwt. | qr. | lb. | oz. |
|---|---|---|---|---|
| | 9 | 3 | 27 | 12 |
| | | | | 7 |
| 3 | 9 | 3 | 26 | 4 |

3. Multiply 9*s* 6*d* by 3? *Ans.* £1 8*s* 6*d*.

4. What will 12 gallons of brandy cost at 9*s* 6*d* per gallon? *Ans.* £5 14*s*.

5. What will 9*cwt.* of butter cost at £1 11*s* 5*d* per *cwt.* *Ans.* £14 2*s* 9*d*.

APPLICATIONS.

1. What is the cost of 4 yards of cloth at £1 3*s* 6*d* per yard?

The amount per yard multiplied by the number of yards will evidently give the entire cost.

OPERATION.

| £ | s | d |
|---|---|---|
| £1 | 3*s* | 6*d* |
| | | 4 |
| £4 | 14*s* | 0*d* *Ans.* |

2. What will be the cost of 9 hats, at 9*s* 9*d* each?
*Ans.* £4 7*s* 9*d*.

3. A farmer has 11 bags of corn each containing 2*bu*. 1*pk*. 3*qt*.: how much corn in all the bags?
*Ans.* 25*bu*. 3*pk*. 1*qt*.

4. What is the cost of 12 bushels of wheat at 9*s* 6*d* per bushel? *Ans.* £5 14*s*.

5. How much sugar in 12 barrels, each containing 3*cwt*. 2*qr*. 27*lb*.? *Ans.* 2*T*. 4*cwt*. 3*qr*. 16*lb*.

6. In 7 loads of wood, each containing 1 cord and 2 cord feet, how many cords? *Ans.* 8 *cords* 6 *cord feet*.

### CASE II.

§ **74**. When the simple number is greater than 12 and a composite number.

### RULE.

*Multiply the denominate number by one of the component parts, or factors, and then multiply the product by the other factors in succession: the last product is the one required.*

### EXAMPLES.

1. Multiply £6 2*s* 9*d* by 48=6×8. *Ans.* £294 12*s*.

2. What will 24 barrels of flour cost, at £2 11*s* 8*d* per barrel? *Ans.* £62.

3. What is the cost of 42*cwt*. of tallow, at £1 14*s* 6*d* per cwt? *Ans.* £72 9*s*.

4. What is the cost of 120 dozen of candles at 5*s* 9*d* per dozen? *Ans.* £34 10*s*.

5. How much water will be contained in 96 hogsheads, each containing 62*gal*. 1*qt*. 1*pt*. 1*gi*.? *Ans.* 5991 *gallons*.

### CASE III.

§ **75**. When the simple number exceeds 12 and is not a composite number.

### RULE.

*Multiply the simple number by each of the denominations separately, and reduce each product to the highest denomination named. Then add the several products together, and their sum will be the answer sought.*

EXAMPLES.

1. Multiply £5 3s 8d by 13.

| | | |
|---|---|---|
| 13 | 13 | 13 |
| 8d | 3s | £5 |
| 104d=8s 8d. | 39s=£1 19s. | £65 |

| | | | |
|---|---|---|---|
| | £65 | | |
| | 1 | 19s | |
| | | 8s | 8d |
| Ans. | £67 | 7s | 8d. |

2. Multiply £6 8s 9d by 139.

| | | | |
|---|---|---|---|
| 139×9d = 1251d= | £ 5 | 4s 3d | |
| 139×8s = 1112s= | £ 55 | 12s | |
| 139×£6=£834 = | £834 | 00 | |
| Ans. | £894 | 16s 3d. | |

3. Multiply £0 2s 4d by 195. *Ans.* £22 15s.

4. What is the cost of 46 bushel of wheat at 4s $7\frac{1}{4}d$ per bushel? *Ans.* £10 11s $9\frac{1}{2}d$.

5. What is the cost of 117*cwt.* of raisins at £1 2s 3d per cwt.? *Ans.* £130 3s 3d.

Q. How do you multiply when the simple number is greater than 12 and a composite number? How do you multiply when the simple number exceeds 12 and is not a composite number?

BILLS OF PARCELS.

A HOSIER'S BILL.

Jan. 4, 1837.

*Mr. Thomas Williams,*

*Bought of Richard Simpson.*

| | | s. | d. | |
|---|---|---|---|---|
| 8 Pair of worsted stockings, | at | 4 | 6 | *per pair.* |
| 5 Pair of thread ditto, - - | at | 3 | 2 | |
| 5 Pair of black silk ditto, - | at | 14 | 0 | |
| 6 Pair of black worsted ditto, | at | 4 | 2 | |
| 4 Pair of cotton ditto, - - | at | 7 | 6 | |
| 2 Yards of fine flannel, - - | at | 1 | 8 | *per yard.* |

Total cost £9 0s 2d.

A MERCER'S BILL.

July 13, 1837.

*Mr. William George,*

*Bought of Peter Thompson.*

| | | | s. | d. | |
|---|---|---|---|---|---|
| 15 | Yards of satin, - - - - | at | 9 | 6 | *per yard.* |
| 18 | Yards of flowered silk, - | at | 17 | 4 | |
| 12 | Yards of rich brocade, - | at | 19 | 8 | |
| 16 | Yards of sarcenet, - - | at | 3 | 2 | |
| 13 | Yards of Genoa velvet. - | at | 27 | 6 | |
| 23 | Yards of lutestring, - - | at | 6 | 3 | |
| | | | | Total Cost | £62 2*s* 5*d*. |

---

A GROCER'S BILL.

Aug. 6, 1837.

*Mr. Nathaniel Parsons,*

*Bought of William Smith.*

| | | | s. | d. | |
|---|---|---|---|---|---|
| 24 | *lb.* of royal green tea, - - | at | 18 | 6 | *per lb.* |
| 24 | *lb.* of imperial tea, - - - | at | 24 | 0 | |
| 35¾ | *lb.* of best bohea, - - - | at | 13 | 10 | |
| 17 | *lb.* of coffee, - - - - - - | at | 5 | 4 | |
| 25 | *lb.* of double refined sugar, - | at | 1 | 1½ | |
| 9 | Sugar loaves, weighing 137*lb.* | at | 0 | 7½ | |
| | | | | Total cost | £85 18*s* 11½*d*. |

## DIVISION OF DENOMINATE NUMBERS.

1. Charles has 3*s* and wishes to divide it equally between himself and two brothers: how much must he give to each? If he divides 2*s* 6*d*, how much? If he divides 2*s*, how much? If he divides 1*s* 6*d*, how much? If he divides 1*s*, how much?

2. John has a bushel of nuts and wishes to divide them equally among himself and three brothers: how much will each have? If he divides 3*pks*., how much? If he divide 2*pks*. 4*qt*.? If he divides 1*pk*.? If he divides 2*qt*.? If he divides 1*qt*.?

3. Divide £25 15*s* 10*d* equally among 8 persons.

In this example we find that 8 is contained in £25, 3 times and £1 over. Now this £1 has yet to be divided by 8, as well as the 15*s* and 10*d*. Then by multiplying the £1 by 20 and adding the the 15*s* gives 35*s*, which contains 8, 4 times and 3*s* over. Multiplying the 3*s* by 12 and adding in the 10*d*, gives 46*d*, which contains 8, 5 times and 6*d* over. The 6*d* being reduced, gives 24 farthings which contains 8, 3 times. Therefore each of the denominations has been divided by 8.

OPERATION.

```
8)£25 15s 10d(£3
   24
  --
 £ 1
   20
  ---
 8)35s(4s
   32
   --
    3s
   12
  ---
 8)46(5d
   40
   --
    6d
    4
  ---
 8)24far.(3far.
```

*Ans.* £3 4*s* 5$\frac{3}{4}$*d*.

§ **76**. Therefore, a denominate number may be divided into any number of equal parts by dividing each of its denominations by the divisor.

## RULE.

I. *Set down the number to be divided in the order of its denominations from the highest to the lowest, and write the divisor on the left.*

II. *Find how often the divisor is contained in the figures of the highest denomination.*

III. *Reduce the remainder, if there be any, to the next lower denomination, and add the figures of the dividend expressing that denomination, and then divide the sum by the divisor.*

IV. *Proceed in the same way for all the denominations, to the last, and if there be a remainder place the divisor under it, as in division of simple numbers. Each of the quotients will be of the same denomination as its dividend, and the several quotients connected together will be the entire quotient sought.*

### PROOF OF MULTIPLICATION.

Divide the product by the multiplier, and if the quotient is equal to the multiplicand, the work may be considered right.

### PROOF OF DIVISION.

Multiply the quotient by the divisor, and if the product is equal to the dividend, the work may be considered right.

**Q.** How may a denominate number be divided? How do you set down the number to be divided? How do you then divide? When there is a remainder what do you do with it? Of what denomination will each of the quotients be? How do you prove multiplication? How do you prove division?

### EXAMPLES.

1. Divide 36*bu.* 3*pk.* 7*qt.* by 7.

In this example we find that 7 is contained in 36 bushels 5 times and 1 bushel over. Reducing this to pecks and adding 3 pecks, gives 7 pecks, which contains 7, 1 time and no remainder. Multiplying 0 by 8 quarts and adding, gives 7 quarts to be divided by 7.

OPERATION.

```
7)36bu. 3pk. 7qt.(5bu.
  35
  --
   1
   4
  ----
7)7pk.(1pk.
  7
  -
  0
  8
7)7(1qt.
Ans. 5bu. 1pk. 1qt.
```

NOTE.—When the divisor does not exceed 12 the division may be made after the manner of short divison in simple numbers.

2. Divide £25 15*s* 4*d* by 8.

We first say 8 into 25, 3 times and £1 or 20*s* over. Then after adding the 15*s*, we say, 8 into 35, 4 times and 3*s* over. Then reducing the 3*s* to pence and adding in the 4*d*, we say 8 into 40, 5 times.

OPERATION.

```
8)£25 15s 4d
  ----------
   £3  4s 5d
```

**Q.** When the divisor does not exceed 12, how may the division be performed?

3. Divide £821 17*s* 9¾*d* by 4. *Ans.* £205 9*s* 5*d* 1¾ *far.*
4. Divide £55 14*s* ¾*d* by 7. *Ans.* £7 19*s* 1*d* 3⅐ *far.*
5. Divide 16*cwt.* 3*qr.* 27*lb.* 6*oz.* by 7.
*Ans.* 2*cwt.* 1*qr.* 19*lb.* 14⅐*oz.*
6. Divide 49*yd.* 3*qr.* 3*na.* by 9. *Ans.* 5*yd.* 2*qr.* ⅞*na.*
7. Divide 131*A.* 1*R.* by 12. *Ans.* 10*A.* 3*R.* 30*P.*
8. Divide £1138 12*s* 4*d* by 53. *Ans.* £21 9*s* 8*d.*
9. Divide 1417*cwt.* 7*lb.* by 79. *Ans.* 17*cwt.* 3*qr.* 21*lb.*
10. Divide £23 15*s* 7¼*d* by 37. *Ans.* 12*s* 10¼*d.*
11. Divide £199 3*s* 10*d* by 53. *Ans.* £3 15*s* 2*d.*

NOTE.—When the divisor is a composite number, and exceeds 12, the work may be shortened by dividing by the factors, in succession, as in division of simple numbers.

EXAMPLES.

1. Divide £28 2*s* 4*d* by the composite number 21. Here the factors are 3 and 7.

| OPERATION. | OPERATION. |
|---|---|
| 7)£28 2*s* 4*d* | 3)£4 0*s* 4*d* |
| £4 0*s* 4*d* | £1 6*s* 9⅓*d.* |

Hence, the answer sought is £1 6*s* 9⅓*d.*

Q. When the divisor is a composite number, how may the division be performed?

2. Divide £57 3*s* 4*d* by 35=5×7. *Ans.* £1 12*s* 8*d.*
3. Divide £85 4*s* by 72. *Ans.* £1 3*s* 8*d.*
4. Divide £31 2*s* 10½*d* by 99. *Ans.* 6*s* 3½*d.*

APPLICATIONS.

1. Bought 65 yards of cloth for which I paid £72 14*s* 4½*d*: what did it cost per yard? *Ans.* £1 2*s* 4½*d.*

2. Bought 64 gallons of brandy for £30 8*s*: what did it cost per gallon? *Ans.* 9*s* 6*d.*

3. Bought 144 reams of paper for £96: what did it cost me per ream? *Ans.* 13*s* 4*d.*

4. Sixty-three barrels of sugar contain 7*T.* 16*cwt.* 3*qr.* 21*lb.*: how much is there in each barrel?
*Ans.* 2*cwt.* 1*qr.* 27*lb*

5. A farmer has a granary containing 232 bushels 3 pecks 7 quarts of wheat, and he wishes to put it in 105 bags: how much will each bag contain? *Ans.* 2*bu.* 7*qt.*

6. One hundred and seventy-six men consumed in a week 13*cwt.* 3*qr.* 1*lb.* 6*oz.* of bread: how much did each man consume? *Ans.* 8*lb.* 12*oz.* 2*dr.*

APPLICATIONS IN THE FOUR RULES.

*Albany, July* 1, 1837.

*Mr. James Sears,*

*Bought of Albert Titus.*

3*lb.* of green tea at 7*s* 6*d* per pound, - -
27*yd.* of muslin at 1*s* 6*d* per yard, - - -
4*cwt.* of sugar at £2 2*s* 8*d* per cwt. - -
2*hhd.* of molasses at 2*s* 6*d* per gallon, - -
6*lb.* of raisins at 1*s* 7*d* per pound. - - -

Received payment, £27 18*s* 2*d.*

*Albert Titus.*

2. A gentleman purchased of a silversmith, 2 dozen silver spoons each weighing 3*oz.* 4*pwt.* 1*gr.*; 2 dozen of tea spoons, each weighing 15*pwt.* 16*gr.*; 3 tankards each weighing 22*oz.* 14*pwt.* He sold him old silver to the amount of 6*lb.* 10*oz.* 3*pwt.*; how much remained to be paid for? *Ans.* 6*lb.* 9*oz.* 12*pwt.*

3. What will be the cost of 22 tons of hay, at £2 1*s* 10*d* per ton? *Ans.* £46 0*s* 4*d.*

4. If two hogsheads of wine cost £67 4*s*: what does it cost per gallon? *Ans.* 10*s* 8*d.*

5. If 4*cwt.* of sugar cost £14: what is it per pound? *Ans.* 7½*d.*

6. A man paid £67 4*s* for a pile of wood containing 64 cords; he sold 30 cords for £29 16*s*: for how much must he sell the remainder per cord so as not to lose? *Ans.* £1 2*s.*

7. If 78*cwt.* 3*qr.* 10*lb.* of sugar be equally divided among 5 men, what will be each one's share? *Ans.* 15*cwt.* 3*qr.* 2*lb.*

8. A printer uses one sheet of paper for every 16 pages of an octavo book: how much paper will be necessary to

print 500 copies of a book containing 336 pages, allowing 2 quires of waste paper in each ream?

*Ans.* 24 *reams* 5 *quires* 12 *sheets.*

9. A farmer wishes to divide 108 acres into 8 equal fields: how much will there be in each field?

*Ans.* 13*A.* 2*R.*

10. Out of a pipe of wine, a merchant draws 12 bottles, each containing 1 pint 3 gills: he then fills six 5 gallon demijohns; then he draws off 3 dozen bottles, each containing 1 quart 2 gills: how much remained in the cask?

*Ans.* 82*gal.* 1*pt.*

11. A man lends his neighbor £135 6*s* 8*d* and takes in part payment 4 cows at £5 8*s* apiece, also a horse worth £50: how much remained due? *Ans.* £63 14*s.* 8*d.*

12. A farmer has 6T. 8*cwt.* 2*qr.* 14*lb.* of hay to be removed in 6 equal loads: how much must be carried at each load? *Ans.* 1T. 1*cwt.* 1*qr.* 21*lb.*

13. A person at his death left landed estate to the amount of £2000, and personal property to the amount of £2803 17*s* 4*d.* He directed that his widow should receive one eighth of the whole, and that the residue should be equally divided among his four children. What was the widow's and each child's portion?

*Ans.* { Widow's portion £600 9*s* 8*d.*
Each child's portion £1050 16*s* 11*d.*

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## OF VULGAR FRACTIONS.

(Before proceeding farther let the pupil study carefully from § 42 to Denominate Numbers.)

§ **77**. There are five kinds of Vulgar Fractions, Proper, Improper, Simple, Compound, and Mixed.

A Proper Fraction is one in which the numerator is less than the denominator. The value of every proper fraction is less than 1. (See § **44**.)

The following are proper fractions:

$$\frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \frac{3}{4},\ \frac{3}{7},\ \frac{5}{8},\ \frac{9}{10},\ \frac{8}{9},\ \frac{5}{6}.$$

AN IMPROPER FRACTION is one in which the numerator is equal to, or exceeds the denominator. Such fractions are called improper fractions because they are equal to, or exceed unity. When the numerator is equal to the denominator the value of the fraction is 1; in every other case the value of an improper fraction is greater than 1.

The following are improper fractions:

$\frac{3}{2}, \frac{5}{3}, \frac{6}{5}, \frac{8}{7}, \frac{9}{8}, \frac{12}{6}, \frac{14}{7}, \frac{19}{7}$.

A SIMPLE FRACTION is a single expression. A simple fraction may be either proper or improper.

The following are simple fractions:

$\frac{1}{4}, \frac{3}{2}, \frac{5}{6}, \frac{8}{7}, \frac{9}{2}, \frac{8}{3}, \frac{6}{3}, \frac{7}{5}$.

A COMPOUND FRACTION is a fraction of a fraction, or several fractions connected together with the word *of* between them.

The following are compound fractions:

$\frac{1}{2}$ of $\frac{1}{4}$, $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{1}{6}$ of 3, $\frac{1}{7}$ of $\frac{1}{8}$ of 4.

A MIXED NUMBER is made up of a whole number and a fraction. The whole numbers are sometimes called *integers*. The following are mixed numbers:

$3\frac{1}{2}, 4\frac{1}{3}, 6\frac{2}{8}, 5\frac{3}{5}, 6\frac{5}{8}, 3\frac{1}{7}$

Q. How many kinds of Vulgar Fractions are there? What are they? What is a proper fraction? Is its value greater or less than 1? What is an improper fraction? Why is it called improper? When is its value equal to 1? What is a simple fraction? What is a compound fraction? What is a mixed number? Give an example of a proper fraction? Of an improper fraction? Of a simple fraction? Of a compound fraction? Of a mixed fraction? Is four-ninths a proper or improper fraction? What kind of a fraction is six-thirds? What is its value? What kind of a fraction is nine-eighths? What is its value? What kind of a fraction is one-half of a third? What kind of a fraction is two and one-sixth? Four and a seventh? Eight and a tenth?

§ 78. The numerator and denominator of a fraction, taken together, are called the *terms* of the fraction. Hence, every fraction has two terms.

Q. What are the terms of a fraction? What are the terms of the fraction three-fourths? Of five-eighths? Of six-sevenths?

§ 79. A whole number may be expressed fractionally by writing 1 below it for a denominator. Thus,

| | | | |
|---|---|---|---|
| 3 may be written | $\frac{3}{1}$ | and is read, | 3 ones. |
| 5 - - - - - | $\frac{5}{1}$ | - - | 5 ones. |
| 6 - - - - - | $\frac{6}{1}$ | - - | 6 ones. |
| 8 - - - - - | $\frac{8}{1}$ | - - | 8 ones. |

But 3 ones are equal to 3, 5 ones to 5, 6 ones to 6, and 8 ones to 8. Hence, the value of a number is not changed by placing 1 under it for a denominator.

Q. How may a whole number be expressed fractionally? Does this alter its value? Give an example.

§ 80. If an apple be divided into 6 equal parts,

| | | |
|---|---|---|
| $\frac{1}{6}$ will express | one of the parts, | |
| $\frac{2}{6}$ - - - | two of the parts, | |
| $\frac{3}{6}$ - - - | three of the parts, | |
| &c. - - - | &c. - - | &c. |

and generally, the denominator shows into how many equal parts the unit is divided, and the numerator how many of the parts are taken.

Hence, also, we may conclude, that,

$\frac{1}{6}\times 2$; that is, $\frac{1}{6}$ taken 2 times $=\frac{2}{6}$,

$\frac{1}{6}\times 3$; that is, $\frac{1}{6}$ taken 3 times $=\frac{3}{6}$,

$\frac{1}{6}\times 4$; that is, $\frac{1}{6}$ taken 4 times $=\frac{4}{6}$;

and consequently we have,

PROPOSITION I. *If the numerator of a fraction be multiplied by any number, the denominator remaining unchanged, the value of the fraction will be increased as many times as there are units in the multiplier.* Hence, *to multiply a fraction by a whole number, we simply multiply the numerator by the number.*

Q. If an apple be divided in six equal parts how do you express one of those parts? Two of them? Three of them? Four of them? Five of them? Repeat the proposition. How do you multiply a fraction by a whole number?

### EXAMPLES.

1. Multiply $\frac{3}{8}$ by 8. *Ans.* $\frac{24}{8}$.
2. Multiply $\frac{7}{5}$ by 5. *Ans.* $\frac{35}{5}$.
3. Multiply $\frac{1}{7}$ by 9. *Ans.* ——.
4. Multiply $\frac{8}{19}$ by 14. *Ans.* $\frac{112}{19}$.
5. Multiply $\frac{7}{6}$ by 20. *Ans.* $\frac{140}{6}$.
6. Multiply $\frac{167}{81}$ by 25. *Ans.* $\frac{4175}{81}$.

§ **81**. If three apples be each divided into 6 equal parts, there will be 18 parts in all; and these parts will be expressed by the fraction $\frac{18}{6}$. If it were required to express but one third of the parts, we should take in the numerator but one third of 18: that is, the fraction $\frac{6}{6}$ would express one third of $\frac{18}{6}$. If it were required to express one sixth of the parts, we should take one sixth of 18, and $\frac{3}{6}$ would be the required fraction.

In each case the fraction $\frac{18}{6}$ has been diminished as many times as there were units in the divisor. Hence,

PROPOSITION II. *If the numerator of a fraction be divided by any number, the denominator remaining unchanged, the value of the fraction will be diminished as many times as there are units in the divisor.* Hence, *a fraction may be divided by a whole number by dividing its numerator.*

Q. If 3 apples be each divided into 6 equal parts, how many parts in all? If 4 apples be so divided, how many parts in all? If 5 apples be so divided, how many parts? How many parts in 6 apples? In 7? In 8? In 9? In 10? What expresses all the parts of the three apples? What expresses one-half of them? One-third of them? One-sixth of them? One-ninth of them? One-eighteenth of them? What expresses all the parts of four apples? One-half of them? One-third of them? One-fourth of them? One-sixth of them? One-eighth of them? One-twelvth of them? One-twenty-fourth of them? Put similar questions for 5 apples, 6 apples, &c. Repeat the proposition. How may a fraction be divided?

### EXAMPLES.

1. Divide $\frac{28}{15}$ by 2, by 7, by 14. *Ans.* $\frac{14}{15}$, $\frac{4}{15}$, $\frac{2}{15}$.
2. Divide $\frac{112}{160}$ by 56, by 28, by 14, by 7. *Ans.* ——.
3. Divide $\frac{400}{117}$ by 25, by 8, by 16, by 4. *Ans.* ——.

§ **82**. Let us again suppose the apple to be divided into 6 equal parts. If now each part be divided into 2 equal parts, there will be 12 parts of the apple, and consequently each part will be but half as large as before.

Three parts in the first case will be expressed by $\frac{3}{6}$, and in the second by $\frac{3}{12}$. But since the parts in the second are only half the parts in the first fraction, it follows that,

$$\tfrac{3}{12} = \text{ one half of } \tfrac{3}{6}.$$

If we suppose the apple to be divided into 18 equal parts, three of the parts will be expressed by $\frac{3}{18}$, and since the parts are but one third as large as in the first case, we have

$$\tfrac{3}{18} = \text{ one third of } \tfrac{3}{6}:$$

and since the same may be said of all fractions, we have

PROPOSITION III. *If the denominator of a fraction be multiplied by any number, the numerator remaining unchanged, the value of the fraction will be diminished as many times as there are units in the multiplier.* Hence, *a fraction may be divided by any number, by multiplying the denominator by that number.*

Q. If an unit be divided in 6 equal parts and then into 12 equal parts, how does one of the last parts compare with one of the first? If the second division be into 18 parts, how do they compare? If into 24? What part of 24 is 6? If the second division be into 30 parts, how do they compare? If into 36 parts? Repeat the proposition. How may a fraction be divided by a whole number?

EXAMPLES.

1. What is $\frac{1}{2}$ of $\frac{1}{4}$. *Ans.* $\frac{1}{8}$.
2. What is $\frac{1}{5}$ of $\frac{3}{7}$. *Ans.* $\frac{3}{35}$.
3. Divide $\frac{3}{16}$ by 4. *Ans.* $\frac{3}{64}$.
4. Divide $\frac{17}{15}$ by 8. *Ans.* $\frac{17}{120}$.
5. Divide $\frac{16}{175}$ by 45. *Ans.* $\frac{16}{7875}$.

§ **83**. If we suppose the apple to be divided into 3 parts instead of 6, each part will be twice as large as before, and three of the parts will be expressed by $\frac{3}{3}$ instead of $\frac{3}{6}$. But this is the same as dividing the denominator 6 by 2; and since the same is true of all fractions, we have

PROPOSITION IV. *If the denominator of a fraction be divided by any number, the numerator remaining unchanged, the value of the fraction will be increased as many times as there are units in the divisor.* Hence, *a fraction may be multiplied by a whole number, by dividing the denominator by that number.*

Q. If we divide 1 apple into three parts and another into 6, how much greater will the parts of the first be, than those of the second? Are the parts larger as you decrease the denominator? If you divide the denominator by 2, how do you affect the parts? If you divide it by 3? By 4? By 5? By 6? By 7? By 8? Repeat the proposition. How may a fraction be multiplied by a whole number?

EXAMPLES.

1. Multiply $\frac{3}{4}$ by 2, by 4. *Ans.* $\frac{3}{2}$, $\frac{3}{1}$.
2. Multiply $\frac{16}{32}$ by 2, 4, 8, 16, 32. *Ans.* $\frac{16}{16}$, $\frac{16}{8}$, $\frac{16}{4}$, $\frac{16}{2}$, $\frac{16}{1}$.
3. Multiply $\frac{8}{48}$ by 2, 4, 6, 8, 12, 16, 24, 48. *Ans.* $\frac{8}{24}$, $\frac{8}{12}$, &c.
4. Multiply $\frac{19}{84}$ by 2, 4, 6, 12, 21, 42. *Ans.* $\frac{19}{42}$, $\frac{19}{21}$, $\frac{19}{14}$, &c., &c.
5. Multiply $\frac{151}{200}$ by 5, 10, 20. *Ans.* $\frac{151}{40}$, $\frac{151}{20}$, $\frac{151}{10}$.

§ **84**. It appears from Prop. I, that if the numerator of a fraction be multiplied by any number, the value of the fraction will be *increased* as many times as there are units in the multiplier. It also appears from Prop. III, that if the denominator of a fraction be multiplied by any number, the value of the fraction will be *diminished* as many times as there are units in the multiplier.

Therefore, when the numerator and denominator of a fraction are both multiplied by the same number, the increase from multiplying the numerator will be just equal to the decrease from multiplying the denominator; hence we have,

PROPOSITION V. *If the numerator and denominator of a fraction be multiplied by the same number, the value of the fraction will remain unchanged.*

Q. If the numerator of a fraction be multiplied by a number, how many times is the fraction increased? If the denominator be multiplied by the same number, how many times is the fraction diminished? If then the numerator and denominator be both multiplied at the same time, is the value changed? Why not? Repeat the proposition.

EXAMPLES.

1. Multiply the numerator and denominator of $\frac{5}{7}$ by 7: this gives $\frac{5}{7}\times\frac{7}{7}=\frac{35}{49}$. *Ans.* $\frac{35}{49}$.

2. Multiply the numerator and denominator of $\frac{10}{12}$ by 3, by 4, by 5, by 6, by 9, by 12, by 15, by 20.

3. Multiply each term of $\frac{125}{102}$ by 7, by 8, by 12, by 14, by 15, by 17, by 45.

§ **85**. It appears from Prop. II, that if the numerator of a fraction be divided by any number, the value of the fraction will be *diminished* as many times as there are units in the divisor. It also appears from Prop. IV, that if the denominator of a fraction be divided by any number, the value of the fraction will be *increased* as many times as there are units in the divisor. Therefore, when the numerator and denominator of a fraction are divided by the same number, the *decrease* from dividing the numerator will be just equal to the *increase* from dividing the denominator: hence we have,

PROPOSITION VI. *If the numerator and denominator of a fraction be divided by the same number, the value of the fraction will remain unchanged.*

Q. If the numerator of a fraction be divided by a number, how many times will the value of the fraction be diminished? If the denominator be divided by the same number, how many times will the value of the fraction be increased? If they are both divided by the same number, will the value of the fraction be changed? Why not? Repeat the proposition.

EXAMPLES.

1. Divide both terms of the fraction $\frac{8}{16}$ by 4: this gives $4)\frac{8}{16}=\frac{2}{4}$. *Ans.* $\frac{2}{4}$.

2. Divide each term by 8: this gives $8)\frac{8}{16}=\frac{1}{2}$.

3. Divide each term of the fraction $\frac{32}{128}$ by 2, by 4, by 8, by 16, by 32.

4. Divide each term of the fraction $\frac{60}{180}$ by 2, by 3, by 4, by 5, by 6, by 10, by 12, by 15, by 20, by 30, by 60.

§ **86**. *Any number greater than unity that will divide two or more numbers without a remainder is called their common divisor: and the greatest number that will so divide them, is called their* GREATEST COMMON DIVISOR.

EXAMPLES.

1. Take the two numbers 142 and 994. The greatest common divisor cannot be greater than the least number 142. This number will divide itself:—let us see if it will also divide 994.

OPERATION.

```
142)142(1
    142
    ---

142)994(7
    994
    ---
```

The number 142 exactly divides itself, giving a quotient of 1; it also divides 994 giving a quotient of 7. Therefore, 142 is the greatest common divisor.

The number 2 and 71 are *common* divisors of the two numbers 142 and 944 since either of them will divide both of the numbers without a remainder. Two numbers may have several common divisors, but they have only one *greatest* common divisor.

Q. What is the common divisor of two or more numbers? What is their greatest common divisor? What is the difference between the common divisor and the greatest common divisor? What is the common divisor of 2 and 4? Of 4 and 6? What are the common divisors of 4 and 8? What is their greatest common divisor? What are the divisors of 12 and 16? Their greatest common divisor?

2. Take the two numbers 72 and 90.

Let us again see if the least number 72, is the greatest common divisor. After dividing we find a remainder of 18.

OPERATION.

```
                          72)90(1
                             72
                             --
greatest common div.      18)72(4
                             72
                             --
```

Now if 18 will divide 72, it will also divide 90, for $90=72+18$, and 18 will be contained once more in $90=72+18$ than in 72: but 18 divides 72 without a remainder: therefore, 18 is the common divisor: hence we see *that the common divisor of two numbers must also be a common divisor between the least number and the remainder after division.* But 18 is the *greatest* common divisor; for, the greatest common divisor must be contained at least *once* more in 90 than in 72: hence, the greatest common divisor cannot be greater than the difference between the two numbers, which, in this case is 18. Therefore, we have

PROPOSITION VII. *The greatest common divisor of two numbers is obtained by dividing the greater by the less, then dividing the divisor by the remainder, and continuing to divide the last divisor by the last remainder until nothing remains. The last divisor will be the greatest common divisor sought.*

Q. Will the common divisor of two numbers divide their remainder after division? How do you find the greatest common divisor of two numbers?

3. Find the greatest common divisor of the two numbers 63 and 81.

```
            OPERATION.                          PROOF.
            63)81(1                             9)63(7
               63                                 63
               --                                 --
               18)63(3                          9)81(9
                  54                              81
                  --                              --
Greatest com. div. 9)18(2
                     18
                     --
```

4. Find the greatest common divisor of 315 and 405. *Ans.* 45.

5. What is the greatest common divisor of the two numbers 2205 and 2835? *Ans.* 315.

6. Find the greatest common divisor of 1157 and 623? *Ans.*

7. Find the greatest common divisor of 792 and 1386? *Ans.* 198.

NOTE. If it be required to find the greatest common divisor of more than two numbers, find first the greatest common divisor of two of them, then of that common divisor and one of the remaining numbers, and so on, for all the numbers: the last common divisor will be the greatest common divisor of all the numbers.

8. What is the greatest common divisor of 246, 372 and 522? *Ans.* 6.

9. What is the greatest common divisor of 492, 744 and 1044? *Ans.* 12.

## LEAST COMMON MULTIPLE.

§ 87. A number is said to be a *common multiple* of two or more numbers, when it can be divided by each of them without a remainder. For example, 6 is a common multiple of 2 and 3, because it is exactly divisible by each of them. So likewise, 12 is a common multiple of 2, 3, 4, and 6. because it is divisible by each of them.

The *least common multiple* of two or more numbers, is the *least* number which they will separately divide without a remainder. For example, 12 is a common multiple of 2 and 3, but it is not the *least* common multiple, since 6 is also divisible by 2 and 3. Now 6 being the least number which is so divisible, it is the least common multiple of 2 and 3.

To find the least common multiple of several numbers, we have the following

### RULE.

I. *Place the numbers on the same line, and divide by the least number that will divide two or more of them without a remainder. and set down in a line below the quotients and the undivided numbers.*

II. *Divide as before, until there is no number greater than 1 that will exactly divide any two of the numbers: then multiply together the numbers of the lower line, and the divisors, and the product will be the least common multiple. If, in comparing the numbers together we find no common divisor, their product is the least common multiple.*

### EXAMPLES.

1. Find the least common multiple of 3, 4 and 8.

OPERATION.

2)3 . . . . 4 . . . 8
2)3 . . . . 2 . . . 4
3 . . . . 1 . . . 2

*Ans.* $2\times1\times3\times2\times2=24.$

We first see, that 2 will divide 4 and 8, but as it will not divide 3, we bring down 3 into the 2nd line: we again see that 2 is a common divisor of 2 and 4; and as there is no common divisor between any two of the numbers of the last line, it follows that $2\times1\times3$ multiplied by the two divisors, is the least common multiple.

**Q. When is one number said to be a common multiple of two or more numbers? Of what numbers is 6 a common multiple? Of what numbers is 8 a common multiple? What is the least common multiple of two or more numbers? What is the difference between a common multiple, and the least common multiple? Give the rule for finding the least common multiple. If the numbers have no common divisor what is the least common multiple?**

2. Find the least common multiple of 3, 8, and 9.

OPERATION.

$$\begin{array}{l} 3)\underline{3 \;\ldots\; 8 \;\ldots\; 9} \\ \;\;\;\underline{1 \;\ldots\; 8 \;\ldots\; 3} \\ 1 \times 8 \times 3 \times 3 = 72. \end{array}$$

We arrange the numbers in a line and see that 3 will divide two of them. We then write down the quotients 1, and 3, and also the 8 which cannot be divided. Then as there is no common divisor between any two of the numbers, 1, 8, and 3, it follows that their product, multiplied by the divisor 3, will give the least common multiple sought.

3. Find the least common multiple of 6, 7, 8 and 10. *Ans.* 840.

4. Find the least common multiple of 21 and 49. *Ans.* 147.

5. Find the least common multiple of 2, 7, 5, 6 and 8. *Ans.* 840.

6. Find the least common multiple of 4, 14, 28 and 98. *Ans.* 196.

7. Find the least common multiple of 13 and 6. *Ans.* 78.

8. Find the least common multiple of 12, 4 and 7. *Ans.* 84.

9. Find the least common multiple of 6, 9, 4, 14 and 16. *Ans.* 1008.

10. Find the least common multiple of 13, 12 and 4. *Ans.* 156.

11. What is the least common multiple of 11, 17, 19, 21 and 7? *Ans.* 74613.

## REDUCTION OF VULGAR FRACTIONS.

§ **88.** Reduction of Vulgar Fractions is the method of changing their forms without altering their value.

A fraction is said to be in its lowest terms, when there is no number greater than 1 that will divide the numerator and denominator without a remainder.

Q. What is reduction? When is a fraction said to be in its lowest terms? Is one-half in its lowest terms? Is two-fourths? Is three-fourths?

CASE I.

§ **89.** To reduce an improper fraction to its equivalent whole or mixed number.

RULE.

*Divide the numerator by the denominator, the quotient will be the whole number; and the remainder, if there be one, placed over the given denominator will form the fractional part.*

EXAMPLES.

1. Reduce $\frac{84}{4}$ and $\frac{67}{9}$ to their equivalent whole or mixed numbers.

| OPERATION. | OPERATION. |
|---|---|
| 4)84 | 9)67 |
| *Ans.* 21 | *Ans.* $7\frac{4}{9}$ |

It was shown in § **44**, that the value of every fraction is equal to the quotient arising from dividing the numerator by the denominator: hence the value of the fraction is not changed by the reduction.

Q. How do you reduce a fraction to its equivalent whole or mixed number? Does this reduction alter its value? Why not? What is four-halves equal to? Eight-fourths? Sixteen-eighths? Twenty-fifths? Thirty-six-sixths? Four-thirds? What is nine-fourths equal to? Four-fifths? Seventeen-sixths? Eighteen-sevenths?

2. Reduce $\frac{99}{8}$ to a whole or mixed number. *Ans.* $12\frac{3}{8}$.

3. In $\frac{19}{7}$ of yards of cloth, how many yards?
*Ans.* $2\frac{5}{7}yd.$

4. In $\frac{51}{9}$ of bushels, how many bushels? *Ans.* $5\frac{6}{9}bu.$

5. If I give $\frac{1}{3}$ of an apple to each one of 15 children, how many apples do I give? *Ans.* 5.

6. Reduce $\frac{327}{125}$, $\frac{3672}{153}$, $\frac{50287}{6941}$, $\frac{987625}{72301}$, to their whole or mixed numbers.

7. If I distribute 878 quarter apples among a number of boys, how many whole apples do I use? *Ans.* $219\frac{2}{4}$.

### CASE II.

§ **90**. To reduce a mixed number to its equivalent improper fraction.

### RULE.

*Multiply the whole number by the denominator of the fraction; to the product add the numerator, and place the sum over the given denominator.*

### EXAMPLES.

1. Reduce $4\frac{4}{5}$ to its equivalent improper fraction.

Here $4\times5=20$: then $20+4=24$; hence,

$\frac{24}{5}$ is the equivalent fraction. *Ans.* $\frac{24}{5}$.

This rule is the reverse of Case I. In the example $4\frac{4}{5}$ we have the integer number 4 and the fraction $\frac{4}{5}$. Now 1 whole thing is equal to 5 fifths, and 4 whole things are equal to 20 fifths; to which, add the 4 fifths, and we obtain the 24 fifths.

Q. How do you reduce a mixed number to its equivalent improper fraction? How many fourths are there in one? In two? In three? How many sixths in four and one-sixth? In eight and two-sixths? In seven and three-sixths? In nine and five-sixths? In ten and five-sixths? How many eighths in two and one-eighth? In three and three-eighths? In four and four-eighths? In five and six-eighths? In seven and seven-eighths? In eight and seven-eighths?

2. Reduce $47\frac{5}{6}$ to its equivalent improper fraction? *Ans.* $\frac{287}{6}$.

3. Reduce $676\frac{37}{51}$, $874\frac{33}{9}$, $690\frac{47}{100}$, $367\frac{9}{104}$, to their equivalent improper fractions.

*Ans.* $\frac{34513}{51}$, $\frac{7899}{9}$, $\frac{69047}{100}$, $\frac{38177}{104}$.

4. Reduce $847\frac{36}{175}$, $874\frac{876}{104}$, $67426\frac{368}{879}$, to their equivalent improper fractions.

5. How many 200ths in $675\frac{187}{200}$ *Ans.* 135187.

6. How many 151ths in $187\frac{41}{151}$. *Ans.* 28278.

### CASE III.

§ **91**. To reduce a fraction to its lowest terms.

### RULE.

I. *Divide the numerator and denominator by any number that will divide them both without a remainder, and then divide* *the quotients arising in the same way until there is no number greater than 1 that will divide them without a remainder.*

II. *Or, find the greatest common divisor of the numerator and denominator and divide them by it. The value of the fraction will not be altered by the reduction.*

EXAMPLES.

1. Reduce $\frac{70}{175}$ to its lowest terms.

1st METHOD.

$\frac{5)70}{5)175}=\frac{7)14}{7)35}=\frac{2}{5}$, which are the lowest terms of the fraction, since no number greater than 1 will divide the numerator and denominator without a remainder.

2nd METHOD, BY THE COMMON DIVISOR.

```
                          70)175(2
                             140
Greatest common div.       35)70(2
                              70
```

$\frac{35)70}{35)175}=\frac{2}{5}$. *Ans.*

Q. When is a fraction in its lowest terms? (see § 88.] How do you reduce a fraction to its lowest terms by the first method? By the second? What are the lowest terms of two-fourths? Of six-eighths? Of nine-twelfths? Of sixteen-thirty-sixths? Of ten-twentieths? Of fifteen-twenty-fourths? Of sixteen-eighteenths? Of nine-eighteenths?

2. Reduce $\frac{104}{312}$ to its lowest terms. *Ans.* $\frac{1}{3}$.
3. Reduce $\frac{1049}{8392}$ to its lowest terms. *Ans.* $\frac{1}{8}$.
4. Reduce $\frac{275}{440}$ to its lowest terms. *Ans.* $\frac{5}{8}$.
5. Reduce $\frac{351}{795}$ to its lowest terms. *Ans.* $\frac{117}{265}$.
6. Reduce $\frac{172}{1118}$ to its lowest terms. *Ans.* $\frac{2}{13}$.
7. Reduce $\frac{63}{81}$ to its lowest terms by the 2nd method. *Ans.* $\frac{7}{9}$.
8. Reduce $\frac{315}{405}$ to its lowest terms by the 2nd method. *Ans.* $\frac{7}{9}$.
9. Reduce $\frac{1157}{623}$ to its lowest terms by the 2nd method. *Ans.* $\frac{13}{7}$.
10. Reduce $\frac{792}{1386}$ to its lowest terms by the 2nd method. *Ans.* $\frac{4}{7}$.

### CASE IV.

§ **92**. To reduce a whole number to an equivalent fraction having a given denominator.

### RULE.

*Multiply the whole number by the given denominator, and set the product over the said denominator.*

### EXAMPLES.

1. Reduce 6 to a fraction whose denominator shall be 4.

Here $6\times4=24$; therefore $\frac{24}{4}$ is the required fraction. It is plain that the fraction will in all cases be equal to the whole number, since it may be reduced to the whole number by Case I.

Q. How do you reduce a whole number to an equivalent fraction having a given denominator? How many thirds in 1? In 2? In 3? In 4? If the denominator be 5, what fraction will you form of 5? Of 4? Of 9? Of 7? Of 8? With the denominator 6, what fraction will you form of 3? Of 4? Of 5? Of 6? Of 7? Of 9?

2. Reduce 15 to a fraction whose denominator shall be 9. *Ans.* $\frac{135}{9}$.

3. Reduce 139 to a fraction whose denominator shall be 175. *Ans.* $\frac{24325}{175}$.

4. Reduce 1837 to a fraction whose denominator shall shall be 181.

5. If the denominator be 837, what fractions will be formed from 327? From 889? From 575?

### CASE V.

§ **93**. To reduce a compound fraction to its equivalent simple one.

### EXAMPLES.

1. Let us take the fraction $\frac{3}{4}$ of $\frac{5}{7}$.

First, $\frac{3}{4}=3\times\frac{1}{4}$: hence the fractions may be written $\frac{3}{4}$ of $\frac{5}{7}=3\times\frac{1}{4}$ of $\frac{5}{7}$; that is, three times one fourth of $\frac{5}{7}$. But $\frac{1}{4}$ of $\frac{5}{7}=\frac{5}{28}$: hence we have,

$$\frac{3}{4} \text{ of } \frac{5}{7}=3\times\frac{5}{28}=\frac{15}{28};$$

a result which is obtained by multiplying together the numerators and denominators of the given fractions.

When the compound fraction consists of more than two simple ones, two of them can be reduced to a simple fraction as above, and then this fraction may be reduced with the next, and so on. We therefore have the following

RULE.

I. *Reduce all mixed numbers to their equivalent improper fractions by Case II.*

II. *Then multiply all the numerators together for a numerator and all the denominators together for a denominator: their products will form the fraction sought.*

2. Reduce $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{5}{7}$ to a simple fraction.

Here $\frac{1}{2}\times\frac{1}{3}\times\frac{5}{7}=\frac{5}{42}$. *Ans.* $\frac{5}{42}$.

3. Reduce $\frac{5}{3}$ of $\frac{3}{6}$ of $\frac{6}{7}$ to a simple fraction.

Here, $\frac{5}{3}\times\frac{3}{6}\times\frac{6}{7}=\frac{90}{126}=\frac{10}{14}=\frac{5}{7}$ by dividing the numerator and denominator of $\frac{90}{126}$, first by 9 and then by 2, as shown in Case III.

Or, $\frac{5}{\not 3}\times\frac{\not 3}{\not 6}\times\frac{\not 6}{7}=\frac{5}{7}$, by cancelling or striking out the 3's and 6's in the numerator and denominator.

By cancelling or striking out the 3's we only divide the numerator and denominator of the fraction by 3; and in cancelling the 6's we divide by 6. *Hence, the value of the fraction is not affected by striking out like figures*, which should always be done when they multiply the numerator and denominator.

4. Reduce $\frac{6}{8}$ of $\frac{8}{9}$ of $\frac{9}{15}$ to a simple fraction.

Here $\frac{6}{8}\times\frac{8}{9}\times\frac{9}{15}=\frac{432}{1080}=\frac{6}{15}=\frac{2}{5}$ *Ans.*

Or, $\frac{6}{\not 8}\times\frac{\not 8}{\not 9}\times\frac{\not 9}{15}=\frac{6}{15}=\frac{2}{5}$ *Ans.*

Q. What is a compound fraction? How do you reduce a compound fraction to a simple one? When you find like figures in the numerator and denominator, what do you do with them? Does this alter the value of the fraction? What is one-half of one-half? One-half of one-third? One-third of one-fourth? One-sixth of one-seventh? Three-halves of one-eighth? Six-thirds of two-ones?

5. Reduce $2\frac{1}{4}$ of $6\frac{1}{2}$ of 7 to a simple fraction.

*Ans.* $\frac{819}{8}=102\frac{3}{8}$.

6. Reduce 5 of $\frac{1}{2}$ of $\frac{1}{7}$ of 6 to a simple fraction.

*Ans.* $\frac{30}{14}=2\frac{1}{7}$.

7. Reduce $6\frac{1}{3}$ of $7\frac{1}{4}$ of $6\frac{31}{27}$ to a simple fraction.

*Ans.* $\frac{106343}{324}$.

CASE VI.

§ 94. To reduce fractions of different denominators to equivalent fractions having a common denominator.

RULE.

I. *Reduce compound fractions to simple ones, and whole or mixed numbers to improper fractions.*

II. *Then multiply each one of the numerators by all the denominators except its own, for the new numerators, and multiply all the denominators together for a common denominator: the common denominator placed under each of the new numerators will form the several fractions sought.*

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{7}{3}$, and $\frac{4}{5}$ to a common denominator.

$1\times3\times5=15$ the new numerator of the 1st.
$7\times2\times5=70$ - - - - - - - - - 2nd.
$4\times3\times2=24$ - - - - - - - - - 3rd.
and $2\times3\times5=30$, the common denominator.

Therefore, $\frac{15}{30}$, $\frac{70}{30}$, and $\frac{24}{30}$, are the equivalent fractions.

It is plain, that this reduction does not alter the values of the several fractions, since the numerator and denominator of each are multiplied by the same number. (See Proposition V.)

When the numbers are small the work may be performed mentally.

Thus, $\frac{1}{2}$ $\frac{1}{4}$ $\frac{2}{5}=\frac{20}{40}$, $\frac{10}{40}$, $\frac{16}{40}$.

Here we find the first numerator by multiplying 1 by 4 and 5; the second, by multiplying 1 by 2 and 5; the third, by multiplying 2 by 4 and 2; and the common denominator by multiplying 2, 4 and 5 together.

Q. What is the first step in reducing fractions to a common denominator? What is the second? Does the reduction alter the values of the several fractions? Why not? When the numbers are small, how may the work be performed?

3. Reduce $2\frac{1}{3}$, and $\frac{1}{2}$ of $\frac{1}{7}$ to a common denominator.

$2\frac{1}{3}=\frac{7}{3}$; and $\frac{1}{2}$ of $\frac{1}{7}=\frac{1}{14}$.

$\frac{7}{3}$ and $\frac{1}{14}=\frac{98}{42}$ and $\frac{3}{42}$; the answers.

4. Reduce $5\frac{1}{2}$, $\frac{6}{7}$ of $\frac{1}{3}$, and 4, to a common denominator.
Ans. $\frac{77}{14}$, $\frac{4}{14}$, and $\frac{56}{14}$.

5. Reduce $\frac{7}{8}$, $\frac{135}{75}$, and 37, to a common denominator.
Ans. $\frac{525}{600}$, $\frac{1080}{600}$, and $\frac{22200}{600}$.

6. Reduce 4, $\frac{31}{25}$, $\frac{62}{2}$, to a common denominator.
Ans. $\frac{200}{50}$, $\frac{62}{50}$, and $\frac{1550}{50}$.

7. Reduce $7\frac{1}{2}$, $\frac{31}{18}$, $6\frac{1}{4}$, to a common denominator.
Ans. $\frac{1080}{144}$, $\frac{248}{144}$, and $\frac{900}{144}$.

8. Reduce $4\frac{1}{9}$, $8\frac{1}{7}$, and $2\frac{1}{2}$ of 5, to a common denominator.
Ans. $\frac{518}{126}$, $\frac{1026}{126}$, $\frac{1575}{126}$.

§ **95**. NOTE 1. It is often convenient to reduce fractions to a common denominator by multiplying the numerator and denominator of each fraction by such a number as shall make the denominators the same in both.

### EXAMPLES.

1. Let it be required to reduce $\frac{1}{2}$ and $\frac{1}{3}$ to a common denominator.

We see at once that if we multiply the numerator and denominator of the first fraction by 3, and the numerator and denominator of the second by 2, that they will have a common denominator.

The two fractions will be reduced to $\frac{3}{6}$ and $\frac{2}{6}$.

2. Reduce $\frac{1}{5}$ and $\frac{1}{3}$ to a common denominator

If we multiply both terms of the first fraction by 3 and both terms of the second by 5, we have

$$\frac{1}{5}=\frac{3}{15}, \text{ and } \frac{1}{3}=\frac{5}{15}.$$

3. Reduce $\frac{1}{6}$, $\frac{1}{12}$, and $\frac{3}{4}$, to a common denominator.
Ans. $\frac{2}{12}$, $\frac{1}{12}$, $\frac{9}{12}$.

4. Reduce $\frac{3}{7}$, $\frac{9}{28}$, $\frac{4}{14}$, to a common denominator.
Ans. $\frac{12}{28}$, $\frac{8}{28}$, $\frac{8}{28}$.

§ **96**. NOTE 2. To reduce fractions to their *least common denominator*, we have the following

### RULE.

I. *Find the least common multiple of the denominators as in* § **87** *and it will be the least denominator sought.*

II. *Multiply the numerator of each fraction by the quotient which arises from dividing the common multiple by the de* *nominator, and the products will be the numerators of the required fractions; under which write the least common denominator.*

EXAMPLES.

1. Reduce $\frac{3}{7}$, $\frac{5}{8}$ and $\frac{2}{6}$ to their least common denominator.

OPERATION.

$$2\underline{)7 - - 8 - - 6}$$
$$\underline{7 - - 4 - - 3}$$

and $3\times4\times7\times2=168$ the least common denominator.

$$\frac{168}{7}\times3=24\times3=72 \text{ 1st numerator}$$

$$\frac{168}{8}\times5=21\times5=105 \text{ 2nd numerator}$$

$$\frac{168}{6}\times2=28\times2=56 \text{ 3rd denominator.}$$

*Ans.* $\frac{72}{168}$, $\frac{105}{168}$, and $\frac{56}{168}$.

2. Reduce $\frac{4}{5}$, $\frac{8}{9}$ and $\frac{3}{15}$ to their least common denominator. *Ans.* $\frac{36}{45}$, $\frac{40}{45}$ and $\frac{9}{45}$.

3. Reduce $14\frac{5}{4}$, $6\frac{3}{8}$ and $5\frac{1}{2}$, to their least common denominator. *Ans.* $15\frac{2}{8}$, $6\frac{3}{8}$ and $5\frac{4}{8}$.

4. Reduce $\frac{3}{15}$, $\frac{4}{24}$ and $\frac{8}{9}$ to their least common denominator. *Ans.* $\frac{72}{360}$, $\frac{60}{360}$, $\frac{320}{360}$.

5. Reduce $\frac{67}{120}$, $\frac{6}{40}$, $\frac{5}{2}$, to their least common denominator. *Ans.* $\frac{67}{120}$, $\frac{18}{120}$, $\frac{300}{120}$.

6. Reduce $\frac{41}{50}$, $3\frac{61}{20}$, $4\frac{1}{2}$ and 8 to a common denominator. *Ans.* $\frac{82}{100}$, $\frac{605}{100}$, $\frac{450}{100}$, $\frac{800}{100}$.

7. Reduce $3\frac{1}{8}$, $4\frac{4}{12}$, $8\frac{6}{18}$, $14\frac{7}{16}$, to their least common denominator. *Ans.* $\frac{450}{144}$, $\frac{624}{144}$, $\frac{1200}{144}$, $\frac{2079}{144}$.

8. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to fractions having the least common denominator. *Ans.* $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$.

9. Reduce $\frac{2}{5}$, $\frac{4}{6}$, $\frac{5}{9}$, and $\frac{7}{10}$ to fractions having the least common denominator. *Ans.* $\frac{36}{90}$, $\frac{60}{90}$, $\frac{50}{90}$, $\frac{63}{90}$.

10. Reduce $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{16}$, and $\frac{17}{24}$ to equivalent fractions having the least common denominator.

*Ans.* $\frac{16}{48}$, $\frac{36}{48}$, $\frac{40}{48}$, $\frac{42}{48}$, $\frac{33}{48}$, $\frac{34}{48}$.

**Q. How do you reduce fractions to their least common denominator? Does this reduction affect the values of the fractions?**

## REDUCTION OF DENOMINATE FRACTIONS.

§ **97**. We have seen § **45**, that a denominate number is one in which the kind of unit is denominated or expressed. For the same reason, a denominate fraction is one which expresses the *kind of unit* that has been divided. Such unit is called the unit of the fraction. Thus, $\frac{2}{3}$ of a £ is a denominate fraction. It expresses that one £ is the unit which has been divided.

The fraction $\frac{3}{8}$ of a shilling is also a denominate fraction, in which the unit that has been divided is one shilling. These two fractions are of different denominations, the unit of the first being one pound, and that of the second, one shilling.

*Fractions, therefore, are of the same denomination when they express parts of the same unit, and of different denominations when they express parts of different units.*

REDUCTION *of denominate fractions consists in changing their denominations without altering their values.*

Q. What is a denominate number? What is a denominate fraction? What is the unit called? In two-thirds of a pound, what is the unit? In three-eighths of a shilling, what is the unit? In one-half of a foot, what is the unit? When are fractions of the same denomination? When of different denominations? Are one-third of a £ and one-fourth of a £ of the same or different denominations? One-fourth of a £ and one-sixth of a shilling? One-fifth of a shilling and one-half of a penny? What is reduction? How many shillings in a £? How many in £2? In 3? In 4? How many pence in 1*s*? In 2? In 3? In 2*s* 8*d*? In 3*s* 6*d*? In 5*s* 8*d*? How many feet in 3 yards 2*ft.*? How many inches?

### CASE I.

§ **98**. To reduce a denominate fraction from a lower to a higher denomination.

### RULE.

I. *Consider how many units of the given denomination make one unit of the next higher, and place* 1 *over that number forming a second fraction.*

II. *Then consider how many units of the second denomination make one unit of the denomination next higher, and place* 1 *over that number forming a third fraction; and so on, to the denomination to which you would reduce.*

III. *Connect all the fractions together, forming a compound fraction; then reduce the compound fraction to a simple one by Case V.*

EXAMPLES.

1. Reduce $\frac{1}{3}$ of a penny to the fraction of a £.

OPERATION.

$\frac{1}{3}$ of $\frac{1}{12}$ of $\frac{1}{20}$ = £$\frac{1}{720}$.

The given fraction is $\frac{1}{3}$ of a penny. But one penny is equal to $\frac{1}{12}$ of a shilling: hence $\frac{1}{3}$ of of a penny is equal to $\frac{1}{3}$ of $\frac{1}{12}$ of a shilling. But one shilling is equal to $\frac{1}{20}$ of a pound: hence $\frac{1}{3}$ of a penny is equal to $\frac{1}{3}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a £ = £$\frac{1}{720}$. The reason of the rule is therefore evident.

2. Reduce $\frac{3}{8}$ of a barleycorn to the denomination of yards.

OPERATION.

$\frac{3}{8}$ of $\frac{1}{3}$ of $\frac{1}{12}$ of $\frac{1}{3} = \frac{3}{864}$ *yards.*

Since 3 barleycorns make an inch, we first place 1 over 3: then as 12 inches make a foot, we place 1 over 12, and as 3 feet make a yard, we next place 1 over 3.

Q. How do you reduce a denominate fraction from a lower to a higher denomination? What is the first step? What the second? What the third?

3. Reduce $\frac{3}{4}$*oz* avoirdupois to the denomination of tons.
*Ans.* $\frac{3}{143360}$T.

4. Reduce $\frac{5}{9}$ of a pint to the fraction of a hogshead.
*Ans.* $\frac{5}{4536}$*hhd.*

5. Reduce $\frac{4}{15}$ of a shilling to the fraction of a £.
*Ans.* £$\frac{4}{300}$.

6. Reduce $\frac{1}{3}$ of a farthing to the fraction of a £.
*Ans.* £$\frac{1}{2880}$.

7. Reduce $\frac{3}{8}$ of a gallon to the fraction of a hogshead.
*Ans.* $\frac{3}{504}$*hhd.*

8. Reduce $\frac{5}{8}$ of a shilling to the fraction of a £.
*Ans.* £$\frac{5}{160}$.

9. Reduce $\frac{187}{127}$ of a minute to the fraction of a day.
*Ans.* $\frac{187}{182880}$*da.*

10. Reduce $\frac{6}{8}$ of a pound to the fraction of a *cwt.*
*Ans.* $\frac{6}{896}$*cwt.*

11. Reduce $\frac{1}{2}$ of an ounce, to the fraction of a ton.
*Ans.* $\frac{1}{71680}$T.

### CASE II.

§ **99**. To reduce a denominate fraction from a higher to a lower denomination.

### RULE.

I. *Consider how many units of the next lower denomination make one unit of the given denomination, and place* 1 *under that number forming a second fraction.*

II. *Then consider how many units of the denomination still lower make one unit of the second denomination and place* 1 *under that number forming a third fraction, and so on, to the denomination to which you would reduce.*

III. *Connect all the fractions together, forming a compound fraction. Then reduce the compound fraction to a simple one by Case V.*

### EXAMPLES.

1. Reduce $\frac{1}{7}$ of a £ to the fraction of a penny.

OPERATION.

$\frac{1}{7}$ of $\frac{20}{1}$ of $\frac{12}{1}=\frac{240}{7}d.$

In this example $\frac{1}{7}$ of a pound is equal to $\frac{1}{7}$ of 20 shillings. But 1 shilling is equal to 12 pence; hence $\frac{1}{7}$ of a £ $=\frac{1}{7}$ of $\frac{20}{1}$ of $\frac{12}{1}=\frac{240}{7}d.$ Hence the reason of the rule is manifest.

Q. What do you first do in reducing a denominate fraction to a lower denomination? What next? What next?

2. Reduce $\frac{4}{7}$*cwt.* to the fraction of a pound. *Ans.* $\frac{448}{7}$*lb.*

3. Reduce $\frac{2}{15}$ of a £ to the fraction of a penny. *Ans.* $\frac{32}{1}$*d.*

4. Reduce $\frac{1}{3}$ of a day to the fraction of a minute. *Ans.* 480*m.*

5. Reduce $\frac{3}{9}$ of an acre to the fraction of a pole. *Ans.* $\frac{480}{9}$*P.*

6. Reduce $\frac{6}{7}$ of a £ to the fraction of a farthing. *Ans.* $\frac{5760}{7}$ *far.*

7. Reduce $\frac{3}{504}$ of a hogshead to the fraction of a gallon. *Ans.* $\frac{3}{8}$*gal.*

8. Reduce $\frac{4}{10}$ of a bushel to the fraction of a pint. *Ans.* $\frac{256}{10}$*pt.*

9. Reduce $\frac{3}{7}$ of a day to the fraction of a second. *Ans.* $\frac{259200}{7}$*sec.*

10. Reduce $\frac{5}{8}$ of a tun to the fraction of a gill.

*Ans.* $\frac{40320}{8}$ *gill.*

CASE III.

**§ 100.** To find the value of a fraction in integers of a less denomination.

RULE.

I. *Multiply the numerator by that number which makes one of the next lower denomination, and divide the product by the denominator.*

II. *If there be a remainder, multiply it by that number which makes one of the denomination still less, and divide again by the denominator. Proceed in the same way to the lowest denomination. The several quotients being connected together, will form the equivalent denominate number.*

EXAMPLES.

1. What is the value of $\frac{2}{3}$ of a £?

We first bring the pounds to shillings. This gives the fraction $\frac{40}{3}$ of shillings, which is equal to 13 shillings and 1 over. Reducing this to pence gives the fraction $\frac{12}{3}$ of pence, which is equal to 4 pence.

OPERATION.

```
  2
 20
3)40
 13s . . . 1 Rem.
           12
         3)12
            4d
Ans. 13s 4d.
```

Q. How much is one-half of a £? One-third of a shilling? One-half of a penny? How much is one-half of a *lb.* Avoirdupois? One-fourth of a ton? One-fourth of a *cwt.*? One-half of a quarter? One-fourth of a quarter? One-seventh of a quarter? One-fourteenth of a quarter? One-twenty-eighth of a quarter? How do you find the value of a fraction in terms of integers of a less denomination?

2. What is the value of $\frac{4}{5}$*lb.* troy? *Ans.* 9*oz.* 12*pwt.*

3. What is the value of $\frac{5}{16}$ of a *cwt.*? *Ans.* 1*qr.* 7*lb.*

4. What is the value of $\frac{5}{8}$ of an acre? *Ans.* 2R. 20P.

5. What is the value of $\frac{1}{6}$ of a £? *Ans.* 3*s* 4*d.*

6. What is the value of $\frac{5}{6}$ of a hogshead?

*Ans.* 52*gal.* 2*qt.*

EXAMPLES.

1. Add $\frac{1}{2}$, $\frac{3}{2}$, $\frac{6}{2}$, and $\frac{3}{2}$ together.

It is evident, since all the parts are halves, that the true sum will be expressed by the number of halves: that is by thirteen two's.

OPERATION.

$1+3+6+3=13.$

Hence, $\frac{13}{2}=$ sum.

Q. When the fractions are of the same denomination and have a common denominator, how do you find their sum? What is the sum of one-third and two-thirds? Of three-fourths, one-fourth, and four-fourths? Of three-fifths, six-fifths, and two-fifths? Of three-sixths, seven-sixths, and nine-sixths? Of one-eighth, three-eighths and four-eighths?

2. Add $\frac{4}{7}$ of a £, $\frac{5}{7}$ of a £, and $\frac{9}{7}$ of a £ together.

*Ans.* $\frac{18}{7}$ £. $=$ £$2\frac{4}{7}$.

3. What is the sum of $\frac{3}{9}+\frac{4}{9}+\frac{6}{9}+\frac{12}{9}+\frac{17}{9}$. *Ans.* $\frac{42}{9}=\frac{14}{3}$.

4. What is the sum of $\frac{3}{14}+\frac{8}{14}+\frac{9}{14}+\frac{5}{14}+\frac{3}{14}$. *Ans.* 2.

CASE II.

§ 107. When the fractions are of the same denomination but have different denominators.

RULE.

*Reduce compound fractions to simple ones, mixed numbers to improper fractions, and all the fractions to a common denominator. Then add them as in Case I.*

EXAMPLES.

1. Add $\frac{6}{2}$, $\frac{4}{3}$, and $\frac{2}{5}$ together.

By reducing to a common denominator the new fractions are

$\frac{90}{30}+\frac{40}{30}+\frac{12}{30}=\frac{142}{30}$,

which, by reducing to the lowest terms becomes $4\frac{11}{15}$.

OPERATION.

$6\times3\times5=90$ 1st numerator.

$4\times2\times5=40$ 2nd numerator.

$2\times3\times2=12$ 3rd numerator.

$2\times3\times5=30$ the denominator.

Q. How do you add fractions which have different denominators? How do you reduce fractions of different denominators to equivalent fractions having a common denominator?

2. Add $\frac{1}{7}$ of a £, $\frac{2}{9}$ of a £, and $\frac{5}{6}$ of a £ together.

*Ans.* £$\frac{453}{378}=$ £$1\frac{75}{378}=$ £$1\frac{25}{126}$.

3. Add [illegible] $\frac{1}{8}$, $4\frac{1}{?}$ and $6\frac{1}{8}$. *Ans.* $10\frac{248}{315}$.

4. Find the least common denominator (see § **96**), and add the fraction $\frac{1}{16}$, $\frac{3}{7}$, $\frac{2}{8}$, and $\frac{4}{9}$. *Ans.* $1\frac{187}{1008}$.

5. Find the least common denominator and add $\frac{6}{12}$, $\frac{3}{5}$, $\frac{4}{8}$, and $\frac{6}{30}$. *Ans.* $1\frac{96}{120}$.

Note. § **105**. When there are mixed numbers, instead of reducing them to improper fractions we may add the whole numbers and the fractional parts separately, and then add their sums.

6. Add $19\frac{1}{7}$, $6\frac{2}{3}$, and $4\frac{4}{5}$ together.

| OPERATION. | OPERATION. |
|---|---|
| Whole numbers. | Fractional parts. |
| $19+6+4=29$. | $\frac{1}{7}+\frac{2}{3}+\frac{4}{5}=\frac{169}{105}=1\frac{64}{105}$. |

Hence, $29+1\frac{64}{105}=30\frac{64}{105}$, the sum.

7. Add $3\frac{1}{4}$, $6\frac{5}{7}$, $8\frac{9}{15}$, and $65\frac{9}{8}$. *Ans.* $84\frac{579}{840}$.

CASE III.

§ **106**. When the fractions are of different denominations.

RULE.

*Reduce the fractions to the same denomination. Then reduce all the fractions to a common denominator, and then add them as in Case I*

EXAMPLES.

1. Add $\frac{2}{3}$ of a £ to $\frac{5}{6}$ of a shilling.

$\frac{2}{3}$ of a £ $=\frac{2}{3}$ of $\frac{20}{1}=\frac{40}{3}$ of a shilling:

Then, $\frac{40}{3}+\frac{5}{6}=\frac{240}{18}+\frac{15}{18}=\frac{255}{18}s=\frac{85}{6}s=14s\ 2d.$

Or, the $\frac{5}{6}$ of a shilling might have been reduced to the fraction of a £ thus,

$\frac{5}{6}$ of $\frac{1}{20}=\frac{5}{120}$ of a £ $=\frac{1}{24}$ of a £.

Then, $\frac{2}{3}+\frac{1}{24}=\frac{48}{72}+\frac{3}{72}=\frac{51}{72}$ of a £: which being reduced by § **100**, gives 14*s* 2*d*. *Ans.* 14*s* 2*d*.

2. Add $\frac{3}{8}$ of a yard to $\frac{5}{9}$ of an inch.

*Ans.* $\frac{253}{648}$*yds.* or $14\frac{1}{18}$*in.*

3. Add $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ of an hour together. *Ans.* 2*da.* $14\frac{1}{2}$*hr.*

4. Add $\frac{4}{7}$ of a *cwt.*, $8\frac{5}{6}$*lb.* and $3\frac{9}{10}$*oz.* together.

*Ans.* 2*qr.* 17*lb.* $1\frac{7}{30}$*oz.*

5. Add $1\frac{1}{4}$ miles, $\frac{7}{10}$ furlongs, and 30 rods together.
*Ans.* 1*m.* 3*fur* 18*rd.*

NOTE. The value of each of the fractions may be found separately, and their several values then added.

6. Add $\frac{3}{5}$ of a year, $\frac{1}{3}$ of a week, and $\frac{1}{8}$ of a day together.

$\frac{3}{5}$ of a year $=\frac{3}{5}$ of $\frac{365}{1}$ days $=219$ days
$\frac{1}{3}$ of a week $=\frac{1}{3}$ of 7 days $=$ 2 days 8 hours
$\frac{1}{8}$ of a day $=$ - - - - 3 hours.
*Ans.* 221*da.* 11*hr.*

7. Add $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{7}{8}$ of a mile together.
*Ans.* 1540*yd.* 2*ft.* 9*in.*

8. Add $\frac{3}{4}$ of a *cwt.*, $\frac{13}{2}$ of a *lb.* 13*oz.* and $\frac{1}{2}$ of a *cwt.* 6*lb.* together. *Ans.* 1*cwt.* 1*qr.* 27*lb.* 13*oz.*

Q. How do you add fractions of different denominations? What is the second method?

## SUBTRACTION OF VULGAR FRACTIONS.

§ **107**. It has been shown (see § **102**), that before fractions can be added together, they must be reduced to the same unit and to a common denominator. The same reductions must be made before subtraction.

SUBTRACTION *of Vulgar Fractions teaches how to take a less fraction from a greater.*

Q. Can one-third of a £ be subtracted from one-third of a shilling without reduction? Can one-fourth of a shilling be subtracted from one-fifth of a shilling? What reductions are necessary before subtraction? What is subtraction?

### CASE I.

§ **108**. When the fractions are of the same denomination and have a common denominator.

### RULE.

*Subtract the less numerator from the greater and place the difference over the common denominator.*

### EXAMPLES.

1. What is the difference between $\frac{5}{8}$ and $\frac{3}{8}$?

Here we have $5-3=2$; hence, $\frac{2}{8}=$the difference.

2. What is the difference between $\frac{125}{365}$ and $\frac{67}{365}$.

*Ans.* $\frac{58}{365}$.

3. From $\frac{335}{105}$ take $\frac{169}{105}$. *Ans.* $\frac{166}{105}$.
4. From $\frac{4978}{9765}$ take $\frac{1697}{9765}$. *Ans.* $\frac{3281}{9765}$.
5. From $\frac{18906}{327}$ take $\frac{909}{327}$. *Ans.* $\frac{17997}{327}$.

CASE II.

§ **109**. When the fractions are of the same denomination, but have different denominators.

RULE.

*Reduce mixed numbers to improper fractions, compound fractions to simple ones, and all the fractions to a common denominator: then subtract them as in Case I.*

EXAMPLES.

1. What is the difference between $\frac{5}{6}$ and $\frac{1}{3}$?

Here, $\frac{5}{6}-\frac{1}{3}=\frac{5}{6}-\frac{2}{6}=\frac{3}{6}=\frac{1}{2}$ answer.

Q. How do you subtract fractions which have the same unit but different denominators? What is the difference between one-half and one-third?

2. What is the difference between $12\frac{1}{2}$ of $\frac{1}{6}$ and 2?

*Ans.* $\frac{1}{12}$.

3. What is the difference between $2\frac{1}{2}$ of a £, and $\frac{3}{15}$ of a £? *Ans.* £2 6*s*.
4. From $\frac{1}{8}$ of 6, take $\frac{16}{17}$ of $\frac{1}{2}$. *Ans.* $\frac{19}{68}$.
5. From $\frac{1}{7}$ of $\frac{3}{6}$ of 7, take $\frac{3}{8}$ of $\frac{5}{4}$. *Ans.* $\frac{1}{32}$.
6. From $37\frac{11}{15}$, take $3\frac{5}{7}$ of $\frac{1}{3}$. *Ans.* $36\frac{52}{105}$.

CASE III.

§ **110**. When the fractions are of different denominations.

RULE.

*Reduce the fractions to the same denomination: then reduce them to a common denominator, after which subtract as in Case I.*

EXAMPLES.

1. What is the difference between $\frac{1}{2}$ of a £, and $\frac{1}{3}$ of a shilling?

$\frac{1}{3}$ of a shilling$=\frac{1}{3}$ of $\frac{1}{20}=\frac{1}{60}$ of a £.

Then, $\frac{1}{2}-\frac{1}{60}=\frac{30}{60}-\frac{1}{60}=\frac{29}{60}$ of a £$=9s\ 8d$.

**Q. How do you subtract fractions which are of different denominations?**

2. What is the difference between $\frac{1}{2}$ of a day and $\frac{2}{3}$ of a second? *Ans.* 11*hr.* 59*m.* $59\frac{1}{3}$*sec.*

3. What is the difference between $\frac{6}{9}$ of a rod and $\frac{3}{7}$ of an inch? *Ans.* 10*ft.* $11\frac{4}{7}$*in.*

4. From $1\frac{3}{4}$ of a *lb.* troy weight, take $\frac{1}{6}$ of an ounce. *Ans.* 1*lb.* 8*oz.* 16*pwt.* 16*gr.*

5. What is the difference between $\frac{4}{15}$ of a hogshead, and $\frac{6}{19}$ of a quart? *Ans.* 16*gal.* 2*qt.* 1*pt.* $3\frac{7}{95}$*gi.*

6. From $\frac{1}{2}$ of a £ take $\frac{3}{4}$ of a shilling? *Ans.* 9*s* 3*d.*

7. From $\frac{3}{5}$*oz.* take $\frac{7}{8}$*pwt.* *Ans.* 11*pwt.* 3*gr.*

8. From $4\frac{3}{7}$*cwt.* take $4\frac{9}{10}$*lb.* *Ans.* 4*cwt.* 1*qr.* 15*lb.* 1*oz.* $9\frac{3}{5}$*dr.*

## MULTIPLICATION OF FRACTIONS.

§ **111**. John gave $\frac{1}{3}$ of a cent for an apple. How much must he give for 2 apples? For 3 apples? For 4? For 5? For 6? For 7? For 8? For 9?

Charles gave $\frac{3}{5}$ of a cent for a peach? How much must he give for 2 peaches? For 3? For 4? For 5? For 6?

### EXAMPLES.

1. Multiply the fraction $\frac{5}{8}$ by 4.

When it is required to multiply a fraction by a whole number, it is required to increase the fraction as many times as there are units in the multiplier, which may be done by multiplying the numerator (see § **80**), or by dividing the denominator (see § **83**).

OPERATION.

$\frac{5}{8}\times4=\frac{20}{8}=\frac{5}{2}=2\frac{1}{2}$;

or by dividing the denominator by 4, we have

$\frac{5}{8}\times4={}_{4)}\frac{5}{8}=\frac{5}{2}=2\frac{1}{2}$.

### CASE I.

§ **112**. To multiply a fraction by a whole number.

### RULE.

*Multiply the numerator, or divide the denominator by the whole number.*

2. Multiply $\frac{37}{144}$ by 12. *Ans.* $3\frac{1}{12}$.
3. Multiply $\frac{47}{49}$ by 7. *Ans.* $6\frac{5}{7}$.
4. Multiply $\frac{175}{47}$ by 9. *Ans.* $\frac{1575}{47}$.
5. Multiply $\frac{127}{15}$ by 5. *Ans.* $42\frac{1}{3}$.
6. Multiply $\frac{369}{145}$ by 49. *Ans.* $124\frac{101}{145}$.

Q. How do you multiply a fraction by a whole number?

§ **113**. Note. When we multiply by a fraction it is required to repeat the multiplicand as many times as there are *units* in the fraction.

For example, to multiply 8 by $\frac{3}{4}$ is to repeat 8, $\frac{3}{4}$ times; that is, to take $\frac{3}{4}$ of 8, which is 6.

Hence, *when the multiplier is less than* 1 *we do not take the whole of the multiplicand, but only such a part of it as the fraction is of unity.* For example, if the multiplier be one half of unity, the product will be half the multiplicand: if the multiplier be $\frac{1}{3}$ of unity, the product will be one third of the multiplicand. Hence, *to multiply by a proper fraction does not imply increase, as in the multiplication of whole numbers.*

Q. What is required when we multiply by a fraction? What is the product of 8 multiplied by one-half? By one-fourth? By one-eighth? By three-halves? By six-halves? What is the product of 9 multiplied by one-half? By one-third? By one-sixth? By one-ninth? When the multiplier is less than 1, how much of the multiplicand is taken? Does the multiplication by a proper fraction imply increase?

### CASE II.

§ **114**. To multiply one fraction by another.

### EXAMPLES.

1. Multiply $\frac{3}{4}$ by $\frac{5}{7}$.

OPERATION.

$$\frac{3}{4}\times\frac{5}{7}=\frac{3}{4}\times5\times\frac{1}{7}=\frac{15}{28}.$$

In this example $\frac{3}{4}$ is to be taken $\frac{5}{7}$ times. That is, $\frac{3}{4}$ is first to be multiplied by 5 and the product divided by 7, a result which is obtained by multiplying the numerators and denominators together.

Hence, we have the following

### RULE.

*Reduce all the mixed numbers to improper fractions, and all compound fractions to simple ones: then multiply the* *numerators together for a numerator, and the denominators together for a denominator.*

**Q.** What is the product of one-sixth by one-seventh? Of three-fourths by one-half? Of six-ninth by three-fifths? Give the general rule for the multiplication of fractions.

2. Multiply $\frac{1}{6}$ of $\frac{3}{7}$ by $8\frac{1}{3}$.

We first reduce the compound fraction to the simple one $\frac{3}{42}$, and then the mixed number to the equivalent fraction $\frac{25}{3}$; after which, we multiply the numerators and denominators together.

OPERATION.

$\frac{1}{6}$ of $\frac{3}{7}=\frac{3}{42}$,

$8\frac{1}{3}=\frac{25}{3}$.

Hence, $\frac{3}{42}\times\frac{25}{3}=\frac{75}{126}=\frac{25}{42}$.

*Ans.* $\frac{25}{42}$.

3. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$. *Ans.* $\frac{21}{24}=\frac{7}{8}$.
4. Multiply $\frac{12}{10}$ by $\frac{3}{4}$ of 9. *Ans.* $8\frac{1}{10}$.
5. Multiply $\frac{1}{8}$ of 3 of $\frac{1}{6}$ by $15\frac{1}{9}$. *Ans.* $\frac{17}{18}$.
6. Multiply $\frac{5}{6}$ by $\frac{2}{3}$ of $\frac{6}{7}$. *Ans.* $\frac{10}{21}$.
7. Required the product of 6 by $\frac{2}{3}$ of 5. *Ans.* 20.
8. Required the product of $\frac{2}{9}$ of $\frac{3}{5}$ by $\frac{5}{8}$ of $3\frac{2}{7}$. *Ans.* $\frac{23}{84}$.
9. Required the product of $3\frac{2}{7}$ by $4\frac{14}{33}$. *Ans.* $14\frac{124}{231}$
10. Required the product of 5, $\frac{2}{3}$, $\frac{2}{7}$ of $\frac{3}{5}$ and $4\frac{1}{6}$. *Ans.* $2\frac{8}{21}$.
11. Required the product of $4\frac{1}{2}$, $\frac{3}{4}$ of $\frac{1}{7}$ and $18\frac{4}{5}$. *Ans.* $9\frac{9}{140}$.
12. Required the product of 14, $\frac{5}{6}$, $\frac{4}{5}$ of 9 and $6\frac{3}{7}$. *Ans.* 540.

§ **115.** NOTE. In multiplying by a mixed number, we may first multiply by the integer, then multiply by the fraction, and then add the two products together. This is the best method when the numerator of the fraction is 1.

EXAMPLES.

1. Multiply 26 by $3\frac{1}{2}$.

We first multiply 26 by 3: the product is 78. Afterwards we multiply 26 by $\frac{1}{2}$: the product is 13: hence the entire product is 91.

OPERATION

$$\begin{array}{r} 26 \\ 3 \\ \hline 78 \\ 26\times\frac{1}{2}=13 \\ \hline 91 \end{array}$$ *Ans.*

2. Multiply 48 by $8\frac{1}{8}$.

We first multiply by 8, and then add an eighth.

**OPERATION.**

$$48 \times 8 = 384$$
$$48 \times \tfrac{1}{8} = \quad 8$$
$$392 \quad \textit{Ans.}$$

3. Multiply 67 by $9\frac{1}{12}$. *Ans.* $608\frac{7}{12}$.
4. Multiply 842 by $7\frac{1}{9}$. *Ans.* $5987\frac{5}{9}$.
5. Multiply 3756 by $3\frac{1}{5}$. *Ans.* $12019\frac{1}{5}$.
6. Multiply 2056 by $5\frac{1}{6}$. *Ans.* $10622\frac{2}{3}$.

**Q.** How may you multiply by a mixed number? When is this the best method?

### APPLICATIONS.

1. What will 7 yards of cloth cost at $\$\frac{3}{4}$ per yard? *Ans.* $\$5\frac{1}{4}$.

2. What will 32 gallons of brandy cost, at $\$1\frac{1}{8}$ per gallon? *Ans.* \$36.

3. If 1*lb.* of tea cost $\$1\frac{1}{4}$, what will $6\frac{1}{4}$*lb.* cost? *Ans.* $\$7\frac{13}{16}$.

4. What will be the cost of $17\frac{1}{2}$ yards of cambric at $2\frac{1}{2}$ shillings per yard? *Ans.* £2 3*s* 9*d*.

5. What will $15\frac{1}{16}$ barrels of cider come to at \$3 per barrel? *Ans.* $\$45\frac{3}{16}$.

6. What will $3\frac{3}{8}$ boxes of raisins cost at $\$2\frac{1}{2}$ per box? *Ans.* $\$8\frac{7}{16}$.

7. What will $15\frac{1}{2}$ barrels of sugar cost at $17\frac{1}{4}$ dollars per barrel? *Ans.* $\$267\frac{3}{8}$.

8. What will $3\frac{3}{4}$ cords of wood cost at $\$3\frac{3}{4}$ per cord? *Ans.* $\$14\frac{1}{16}$.

## DIVISION OF VULGAR FRACTIONS.

§ **116.** We have seen that division of integer numbers explains the manner of finding how many times a less number is contained in a greater.

In division of fractions, the divisor may be larger than the dividend, in which case the quotient will be less than 1.

For example, divide 1 apple into 4 equal parts.

Here it is plain that each part will be $\frac{1}{4}$; or that the dividend will contain the divisor but $\frac{1}{4}$ times.

Again, divide $\frac{1}{2}$ of a pear into 6 equal parts.

If a whole pear were divided into 6 equal parts each part would be expressed by $\frac{1}{6}$. But since the half of the pear was divided, each part will be expressed by $\frac{1}{2}$ of $\frac{1}{6}$ or $\frac{1}{12}$.

When we divide a fraction by a whole number we are to divide the fraction into as many equal parts as there are units in the divisor, and this may be done by dividing the numerator as in § **81**, or by multiplying the denominator as in § **82**.

Q. What does division of whole numbers explain? In division of fractions may the divisor exceed the dividend? How will the quotient then compare with 1? If an apple be divided in 2 equal parts, what will express each part? If half an apple be divided into 4 equal parts what will express one of the parts? What is one-half of one-half? What is one-sixth of one-half? One-sixth of one-fourth? One-seventh of three-fourths? One-eighth of one-half? One-ninth of one-third? One-tenth of two-thirds?

CASE I.

§ **117**. To divide a fraction by a whole number.

RULE.

*Divide the numerator or multiply the denominator by the whole number.*

EXAMPLES.

1. Divide $\frac{4}{3}$ by 2.

In the first operation we divide the fraction by multiplying the denominator: in the second we divide the numerator, giving the same result in both cases.

OPERATION.

$$\frac{4}{3}\div 2=\frac{4}{3\times 2}=\frac{4}{6}=\frac{2}{3}.$$

$$\text{or } \frac{4}{3}\div 2=\frac{2)4}{3}=\frac{2}{3}.$$

2. Divide $\frac{18}{37}$ by 9. *Ans.* $\frac{18}{333}=\frac{2}{37}$.
3. Divide $\frac{405}{19}$ by 15. *Ans.* $\frac{405}{285}=\frac{27}{19}$.
4. Divide $\frac{2755}{3758}$ by 19. *Ans.* $\frac{145}{3758}$.
5. Divide $\frac{379}{1267}$ by 15. *Ans.* $\frac{379}{19005}$.

Q. How do you divide a fraction by a whole number?

CASE II.

## § 118. To divide one fraction by another.

EXAMPLES.

1. Divide $\frac{10}{24}$ by $\frac{5}{8}$.

1st OPERATION.

$\frac{5}{8}=5\times\frac{1}{8}$

$\frac{10}{24}\div5=\frac{10}{120}$

$\frac{10}{120}\times8=\frac{80}{120}=\frac{2}{3}$.

If the divisor were 5 the quotient would be $\frac{10}{120}$. But, since the divisor is $\frac{1}{8}$ of 5 the true quotient must be 8 times $\frac{10}{120}$, for the eighth of a number will be contained in the dividend 8 times more than the number itself. In this operation we have actually multiplied the numerator of the dividend by 8 and the denominator by 5: that is, we have, *inverted the terms of the divisor and multiplied the fractions together.*

2nd OPERATION.

$\frac{10}{24}\div\frac{5}{8}=\frac{5)10}{8)24}=\frac{2}{3}$.

Since multiplying the denominator by 5 is the same as dividing the numerator, and multiplying the numerator the same as dividing the denominator, we may, if we please, divide 10 by 5 and 24 by 8.

Hence, for the division of vulgar fractions we have the following

RULE.

*Reduce compound fractions to simple ones, and whole numbers to improper fractions; then divide the numerator by the numerator and the denominator by the denominator, if they will exactly divide: but if not, invert the terms of the divisor, and then multiply the divisor and dividend together.*

Q. How do you divide one vulgar fraction by another?

EXAMPLES.

1. Divide $\frac{16}{90}$ by $\frac{8}{5}$.

Here we divide the numerator by the numerator, and the denominator by the denominator.

1st OPERATION.

$\frac{8)16}{5)90}=\frac{2}{18}=\frac{1}{9}$.

Here, we have inverted the terms of the divisor and multiplied the fractions together.

2nd OPERATION.

$\frac{16}{90}\div\frac{8}{5}$ is equal to

$\frac{16}{90}\times\frac{5}{8}=\frac{80}{720}=\frac{1}{9}$.

2. Divide $\frac{1}{2}$ by $\frac{1}{16}$. *Ans.* 8.

3. Divide $\frac{1}{3}$ by $\frac{1}{16}$. *Ans.* $5\frac{1}{3}$.

In the division of fractions we should note the following principles.

1st. When the dividend is just equal to the divisor, the quotient will be 1.

2nd. When the dividend is greater than the divisor, the quotient will be greater than 1.

3rd. When the dividend is less than the divisor, the quotient will be less than 1.

4th. The quotient will be just so many times greater than 1, as the dividend is greater than the divisor.

5th. The quotient will be just as many times less than 1, as the dividend is less than the divisor.

*Q.* How do you divide one fraction by another? When will the quotient be 1? When greater than 1? When less than 1? When greater than 1, how many times greater? When less than 1, how many times less?

4. Divide $\frac{1}{8}$ by $\frac{1}{7}$. *Ans.* $\frac{7}{8}$

5. Divide $3\frac{1}{4}$ by $\frac{1}{9}$. *Ans.* $29\frac{1}{4}$

6. Divide $16\frac{1}{2}$ of $\frac{1}{3}$ by $4\frac{1}{7}$. *Ans.* $1\frac{19}{58}$.

7. Divide $44\frac{1}{33}$ by $\frac{213}{333}$. *Ans.* $68\frac{5877}{7029}$.

8. Divide $371\frac{1}{2}$ by $1\frac{1}{414}$. *Ans.* $370\frac{251}{415}$.

9. Divide $\frac{64}{111}$ by $\frac{23}{1213}$. *Ans.* $30\frac{1042}{2553}$.

10. Divide $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$. *Ans.* $\frac{2}{3}$.

11. Divide 5 by $\frac{7}{10}$. *Ans.* $7\frac{1}{7}$.

12. Divide $5205\frac{1}{5}$ by $\frac{4}{5}$ of 91. *Ans.* $71\frac{1}{2}$.

13. Divide 100 by $4\frac{7}{8}$. *Ans.* $20\frac{20}{39}$.

14. Divide $\frac{3}{4}$ of $\frac{7}{8}$ by $\frac{2}{3}$. *Ans.* $\frac{63}{64}$.

15. Divide $\frac{5}{6}$ of 50 by $4\frac{1}{3}$. *Ans.* $9\frac{8}{13}$.

16. Divide $14\frac{1}{8}$ of $\frac{1}{9}$ by $3\frac{1}{3}$ of 6. *Ans.* $\frac{113}{1440}$.

APPLICATIONS.

1. If 7*lb.* of sugar cost $\frac{44}{75}$ of a dollar, what is the price per pound?

$\frac{44}{75} \div 7 = \frac{44}{525}$ of \$1; or $\frac{44}{525}$ of $\frac{100}{1}$ cents $= \frac{4400}{525} = 8\frac{8}{21}$.

*Ans.* $8\frac{8}{21}$ *cents.*

2. If $\frac{3}{7}$ of a dollar will pay for $10\frac{1}{2}$*lb.* of nails, how much is the price per pound? *Ans.* $\$\frac{2}{49}=4\frac{4}{49}$*cts.*

3. If $\frac{4}{7}$ of a yard of cloth cost \$3, what is the price per yard? *Ans.* $\$5\frac{1}{4}$.

4. If $\$21\frac{1}{2}$ will buy $7\frac{13}{16}$ barrels of apples, how much are they per barrel? *Ans.* $\$2\frac{94}{125}$.

5. If $4\frac{1}{7}$ gallons of molasses cost $\$2\frac{5}{9}$, how much is it per quart? *Ans.* $15\frac{110}{261}$*cts.*

6. If $1\frac{1}{8}$*hhd.* of wine cost $\$250\frac{1}{3}$, how much is the wine per quart? *Ans.* $\$\frac{1502}{1701}=88\frac{512}{1701}$*cts.*

7. If eight pounds of tea cost $7\frac{5}{8}$ of a dollar, how much is it per pound? *Ans.* $95\frac{5}{16}$*cts.*

8. In $8\frac{1}{2}$ weeks a family consumes $165\frac{3}{9}$ pounds of butter: how much do they consume a week? *Ans.* $19\frac{69}{153}$*lb.*

9. If a piece of cloth containing $176\frac{3}{4}$ yards costs $\$375\frac{6}{5}$ what does it cost per yard? *Ans.* $\$2\frac{454}{3535}$.

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## DECIMAL FRACTIONS.

§ **119**. If the unit 1 be divided into 10 equal parts the parts are called *tenths,* because each part is one-tenth of unity.

If the unit 1 be divided into one hundred equal parts, the parts are called *hundredths*, because each part is one hundredth of unity.

If the unit 1 be divided into one thousand equal parts, the parts are called *thousandths*, because each part is one thousandth of unity: and we have similar expressions for the parts, when the unit is divided into ten thousand, one hundred thousand, &c., equal parts.

The division of the unit into tenths, hundredths, thousandths, &c., forms a system of numbers called *Decimal Fractions.*

DECIMAL FRACTIONS.

| | |
|---|---|
| Four tenths, - - - - - - - - - - - - | $\frac{4}{10}$. |
| Six tenths, - - - - - - - - - - - - | $\frac{6}{10}$. |
| Forty-five hundredths, - - - - - - - - | $\frac{45}{100}$. |
| 125 thousandths, - - - - - - - - - - | $\frac{125}{1000}$. |
| 1047 ten thousandths, - - - - - - - - | $\frac{1047}{10000}$. |

From which we see, that in each case the denominator gives denomination or name to the fraction; that is, determines whether the parts are tenths, hundredths, thousandths, &c.

Q. When the unit 1 is divided into 10 equal parts, what is each part called? What is each part called when it is divided in 100 equal parts? When into 1000? Into 10,000, &c.? How are decimal fractions formed? What gives denomination to the fraction?

§ **120**. The denominators of decimal fractions are seldom set down. The fractions are usually expressed by means of a point, or comma, placed at the left of the numerator.

| | | |
|---|---|---|
| Thus, four tenths, | - - - - | ,4 |
| forty-five hundredths, | - - | ,45 |
| 125 thousandths, | - - - | ,125 |
| 1047 ten thousandths, | - - | ,1047 |

The denominator of every decimal fraction, however, is always understood. *It is a unit* 1 *with as many ciphers annexed as there are places of figures in the numerator.*

The place next to the decimal point is called tenths; the next place to the right, the place of hundredths; the next, the place of thousandths; and so on for places further to the right, according to the following Table.

DECIMAL NUMERATION TABLE.

| Tenths. | Hundredths. | Thousandths. | Tens of thousandths. | Hundreds of thousandths. | Millionths. | Tens of millionths. | | |
|---|---|---|---|---|---|---|---|---|
| ,4 | | | | | | | is read | 4 tenths. |
| ,6 | 4 | | | | | | - - | 64 hundredths. |
| ,0 | 6 | 4 | | | | | - - | 64 thousandths. |
| ,6 | 7 | 5 | 4 | | | | - - | 6754 ten thousandths. |
| ,0 | 1 | 2 | 3 | 4 | | | - - | 1234 hundred thousandths. |
| ,0 | 0 | 7 | 6 | 5 | 4 | | - - | 7654 millionths. |
| ,0 | 0 | 4 | 3 | 6 | 0 | 4 | - - | 43604 ten millionths. |

Decimal fractions are numerated from the left hand to the right, beginning with the tenths, hundredths, &c., as in the table.

Q. Are the denominators of decimal fractions generally set down? How are the fractions expressed? Is the denominator understood? What is it? What is the place next the decimal point called? The next? The third, &c.? Which way are decimals numerated?

§ **121.** Let us now write and numerate the following decimals.

| | |
|---|---|
| Four tenths, - - - - - | ,4. |
| Four hundredths, - - - | ,0 4. |
| Four thousandths, - - - | ,0 0 4. |
| Four ten thousandths, - - - | ,0 0 0 4. |
| Four hundred thousandths, - - | ,0 0 0 0 4. |
| Four millionths, - - - - | ,0 0 0 0 0 4. |
| Four ten millionths, - - - | ,0 0 0 0 0 0 4. |

Here we see, that the same figure expresses different values, according to the place which it occupies.

But $\frac{1}{10}$ of $\frac{4}{10}$ is equal to $\frac{4}{100}=$,04.
- $\frac{1}{10}$ of $\frac{4}{100}$ - - - $\frac{4}{1000}=$,004.
- $\frac{1}{10}$ of $\frac{4}{1000}$ - - - $\frac{4}{10000}=$,0004.
- $\frac{1}{10}$ of $\frac{4}{10000}$ - - - $\frac{4}{100000}=$,00004.
- $\frac{1}{10}$ of $\frac{4}{100000}$ - - - $\frac{4}{1000000}=$,000004.
- $\frac{1}{10}$ of $\frac{4}{1000000}$ - - - $\frac{4}{10000000}=$,0000004.

*Therefore, the value of the places diminish in a tenfold proportion from the left hand to the right.*

Hence, ten of the parts in any one of the places, are equal to one of the parts in the place next to the left; that is, ten thousandths make one hundreth, ten hundreths make one tenth, and ten tenths a unit 1.

This law of increase from the right hand towards the left, is the same as in whole numbers. *Therefore, whole numbers and decimal fractions may be united by placing the decimal point between them.*

| | | | |
|---|---|---|---|
| Thus, | 36,95 | - | thirty-six, and 95 hundreths. |
| | 127,4 | - | 127 and four tenths. |
| | 163,03 | - | 163 and three hundredths. |
| | 627,0047 | - | 627 and 47 ten thousandths. |

A number composed partly of a whole number and partly of a decimal, is called a mixed number.

Q. Does the value of a figure depend upon the place which it occupies? How does the value change from the left towards the right? What do ten parts of any one place make? How do they increase from the right towards the left? How may whole numbers be joined with decimals? What is a number called when composed partly of whole numbers and partly of decimals?

Write the following numbers in figures, and numerate them.

1. Forty one, and three tenths. 41,3.
2. Sixteen, and three millionths. 16,000003.
3. Five, and nine hundredths. 5,09.
4. Sixty-five, and fifteen thousandths. 65,015.
5. Eighty, and three millionths. 80,000003.
6. Two, and three thousand millionths. 2,000000003.
7. Four hundred and ninety-two thousandths. 0,492.
8. Three thousand, and twenty-one ten thousandths.
9. Forty seven, and twenty-one ten thousandths.
10. Fifteen hundred and three millionths.
11. Thirty-nine, and six hundred and forty thousandths.
12. Three thousand, eight hundred and forty millionths.
13. Six hundred and fifty thousandths.
14. Fifty thousand, and four hundredths.
15. Six hundred, and eighteen ten thousandths.
16. Three millionths.
17. Thirty nine hundred thousandths.

§ **122**. The denominations of Federal Money will correspond to the decimal division, if we regard 1 dollar as the unit. For, the dimes are tenths of the dollar, the cents are hundredths of the dollar, and the mills, being tenths of the cent, are thousandths of the dollar.

EXAMPLES.

1. Express $16, 3 dimes 8 cents and 9 mills decimally. *Ans.* $16,389.
2. Express $95, 8 dimes 9 cents 5 mills decimally. *Ans.* $95,895.
3. Express $107, 9 dimes 6 cents 8 mills decimally. *Ans.* $107,968

4. Express \$47 and 25 cents decimally. *Ans.* \$47,25.
5. Express \$39,39 cents and 7 mills decimally.
*Ans.* \$39,397.
6. Express \$12 and 3 mills decimally. *Ans.* \$12,003.
7. Express \$147 and 4 cents decimally. *Ans.* \$147,04.
8. Express \$148, 4 mills decimally. *Ans.* \$148,004.
9. Express four dollars, six mills decimally. *Ans.* \$4,006.

Q. If the denominations of Federal Money be expressed decimally, what is the unit? What part of a dollar is 1 dime? What part of a dime is a cent? What part of a cent is a mill? What part of a dollar is 1 cent? 1 mill?

§ **123**. A cipher is annexed to a number, when it is placed on the right of it. If ciphers be annexed to the numerator of a decimal fraction, the same number of ciphers must also be annexed to the denominator; for there must be as many ciphers in the denominator as there are places of figures in the numerator (see § **120**.) The numerator and denominator will therefore be multiplied by the same number, and consequently the value of the fraction will not be changed (see § **84**.) Hence,

*Annexing ciphers to a decimal fraction does not alter its value.*

We may take as an example $,3=\frac{3}{10}$. If now we annex a cipher to the numerator, we must, at the same time, annex one to the denominator, which gives

$,30 = \frac{30}{100}$ by annexing one cipher,
$,300 = \frac{300}{1000}$ by annexing two ciphers,
$,3000=\frac{3000}{10000}$ all of which are equal to $\frac{3}{10}=,3$.

Also, $,5= \frac{5}{10}=,50=\frac{50}{100}=,500=\frac{500}{1000}$.

Also, $,8=,80=,800=,8000=,80000$.

Q. When is a cipher annexed to a number? Does the annexing of ciphers to a decimal alter its value? Why not? What does three-tenths become by annexing a cipher? What by annexing two ciphers? Three ciphers? What does ,8 become by annexing a cipher? By annexing two ciphers? By annexing three ciphers?

§ **124**. Prefixing a cipher is placing it on the left of a number. If ciphers be prefixed to the numerator of a decimal fraction, that is, placed at the left hand of the significant figures, the same number of ciphers must be annexed to the denominator. Now, the numerator will remain unchanged while the denominator will be increased ten times for every cipher which is annexed, and the value of the fraction will be decreased in the same proportion (see § **82**). Hence,

*Prefixing ciphers to a decimal fraction diminishes its value ten times for every cipher prefixed.*

Take the fraction $,2=\frac{2}{10}$ as an example.

$,02=\frac{02}{100}$ by prefixing one cipher:

$,002=\frac{002}{1000}$ by prefixing two ciphers:

$,0002=\frac{0002}{10000}$ by prefixing three ciphers:

in which the fraction is diminished ten times for every cipher prefixed.

Also, ,03 becomes ,003 by prefixing one cipher; and ,0003 by prefixing two.

**Q.** When is a cipher prefixed to a number? When prefixed to a decimal, does it increase the numerator? Does it increase the denominator? What effect then has it on the value of the fraction? What does ,5 become by prefixing a cipher? By prefixing two ciphers? By prefixing three? What does ,07 become by prefixing a cipher? By prefixing two? By prefixing three? By prefixing four?

## ADDITION OF DECIMAL FRACTIONS.

§ **125**. It must be recollected that only like parts of unity can be added together, and therefore in setting down the numbers for addition the figures occupying places of the same value must be placed directly under each other.

The addition of decimal fractions is then made in the same manner as that of whole numbers.

Add 37,04, 704,3 and ,0376 together.

In this example, we place the tenths under tenths, the hundredths under hundredths, and this brings the decimal points and the like parts of the unit directly under each other. We then add as in whole numbers.

OPERATION.

```
  37,04
 704,3
    ,0376
---------
 741,3776
```

Hence, for addition of decimals we have the following

RULE.

I. *Set down the numbers to be added so that tenths shall fall under tenths, hundredths under hundredths, &c. This will bring all the decimal points directly under each other.*

II. *Then add as in simple numbers and point off in the sum, from the right hand, so many places for decimals as are equal to the greatest number of places in any of the given numbers.*

Q. What parts of unity may be added together? How do you set down the numbers for addition? How will the decimal points fall? How do you then add? How many decimal places do you point off in the sum?

EXAMPLES.

1. Add 4,035, 763,196, 445,3741 and 91,3754 together. *Ans.* 1303,9805.

2. Add 365,103113, ,76012, 1,34976, ,3549 and 61,11 together. *Ans.* 428,677893.

3. 67,407+97,004+4+,6+,06+,3=169,371.

4. ,0007+1,0436+,4+,05+,047=1,5413.

5. ,0049+47,0426+37,0410+360,0039=444,0924.

6. Required the sum of twenty-nine and 3 tenths, four hundred and sixty-five, and two hundred and twenty-one thousandths. *Ans.* 494,521.

7. Required the sum of two hundred dollars one dime three cents and nine mills, four hundred and forty dollars nine mills, and one dollar one dime and one mill.

*Ans.* $641,249, or 641 dollars 2 dimes 4 cents 9 mills.

8. What is the sum of one tenth, one hundredth, and one thousandth. *Ans.* ,111.

9. What is the sum of 4, and 6 ten thousandths.

*Ans.* 4,0006.

10. Required in dollars and decimals, the sum of one dollar one dime one cent one mill, six dollars three mills, four dollars eight cents, nine dollars six mills, one hundred dollars six dimes, nine dimes one mill, and eight dollars six cents. *Ans.* $129,761.

11. What is the sum of 4 dollars 6 cents, 9 dollars 3 mills, 14 dollars 3 dimes 9 cents 1 mill, 104 dollars 9 dimes 9 cents 9 mills, 999 dollars 9 dimes 1 mill, 4 mills, 6 mills, and 1 mill. *Ans.* $1132,365.

## SUBTRACTION OF DECIMAL FRACTIONS.

§ **126**. Subtraction of Decimal Fractions teaches how to find the difference between two decimal numbers.

### EXAMPLES.

1. From 3,275 take ,0879.

In this example a cipher is annexed to the minuend to make the number of decimal places equal to the number in the subtrahend. This does not alter the value of the minuend (see § **123**).

OPERATION.

```
3,2750
 ,0879
3,1871
```

Hence, we have the following

### RULE.

I. *Set down the less number under the greater, so that figures occupying places of the same value shall fall directly under each other.*

II. *Then subtract as in simple numbers, and point off in the remainder as many places for decimals as are equal to the greatest number of places in either of the given numbers.*

Q. What does subtraction teach? How do you set down the numbers for subtraction? How do you then subtract? How many decimal places do you point off in the remainder?

2. From 3295 take ,0879. *Ans.* 3294,9121.
3. From 291,10001 take 41,375. *Ans.* 249,72501.
4. From 10,000001 take ,111111. *Ans.* 9,888890.
5. From three hundred and ninety-six, take 8 ten thousandths. *Ans.* 395,9992.
6. From 1 take one thousandth. *Ans.* ,999.
7. From 6378 take one tenth. *Ans.* 6377,9.
8. From 365,0075 take 3 millionths. *Ans.* 365,007497.
9. From 21,004 take 97 ten thousandths. *Ans.* 20,9943.
10. From 260,4709 take 47 ten millionths. *Ans.* 260,4708953.
11. From 10,0302 take 19 millionths. *Ans.* 10,030181.
12. From 2,01 take 6 ten thousandths. *Ans.* 2,0094.

## MULTIPLICATION OF DECIMAL FRACTIONS.

EXAMPLES.

§ 127. 1. Multiply ,37 by ,8.

If we multiply the fraction $\frac{37}{100}$ by $\frac{8}{10}$, we find the product to be $\frac{296}{1000}$: and generally, the number of ciphers in the denominator of the product, will be equal to the number of decimal places in the two factors.

OPERATION.

$$,37 = \frac{37}{100}.$$
$$,8 = \frac{8}{10}.$$
$$,296 = \frac{296}{1000}.$$

2. Multiply ,3 by ,02.

OPERATION.

$$,3 \times ,02 = \frac{3}{10} \times \frac{02}{100} = \frac{6}{1000} = ,006 \text{ answer.}$$

To express the 6 thousandths decimally we have to prefix two ciphers to the 6, and this makes as many decimal places in the product as there are in both multi plicand and multiplier.

Therefore, to multiply one decimal by another, we have the following

RULE.

*Multiply as in simple numbers, and point off in the product, from the right hand, as many figures for decimals as are equal to the number of decimal places in the multiplicand and multiplier; and if there be not so many in the product, supply the deficiency by prefixing ciphers.*

Q. After multiplying, how many decimal places will you point off in the product? When there are not so many in the product, what do you do? Give the rule for the multiplication of decimals.

EXAMPLES.

1. Multiply 3,049 by ,012. *Ans.* ,036588.

| (2.) | | (3.) | |
|---|---|---|---|
| Multiply | 365,491 | Multiply | 496,0135 |
| by | ,001 | by | 1,496 |
| *Ans.* | ,365491 | *Ans.* | 742,0361960 |

4. Multiply one and one millionth by one thousandth.
*Ans.* ,001000001.

5. Multiply one hundred and forty-seven millionths, by one millionth. *Ans.* ,000000000147.

6. Multiply three thousand, and twenty-seven hundredths by 31. *Ans.* 9308,37.

7. Multiply 31,00467 by 10,03962.
*Ans.* 311,2751050254.

8. What is the product of five-tenths by five-tenths.
*Ans.* ,25.

9. What is the product of five-tenths by five thousandths. *Ans.* ,0025.

10. Multiply 596,04 by 0,00004. *Ans.* ,0238416.

11. Multiply 38049,079 by 0,00008. *Ans.* 3,04432632.

§ **128**. NOTE. When a decimal number is to be multiplied by 10, 100, 1000, &c., the multiplication may be made by removing the decimal point as many places to the right hand as there are ciphers in the multiplier, and if there be not so many figures on the right of the decimal point, supply the deficiency by annexing ciphers

$$\text{Thus, } 6{,}79 \text{ multiplied by} \left\{\begin{array}{l} 10 \\ 100 \\ 1000 \\ 10000 \\ 100000 \end{array}\right\} = \left\{\begin{array}{r} 67{,}9 \\ 679, \\ 6790, \\ 67900, \\ 679000, \end{array}\right.$$

$$\text{Also, } 370{,}036 \text{ multiplied by} \left\{\begin{array}{l} 10 \\ 100 \\ 1000 \\ 10000 \\ 100000 \end{array}\right\} = \left\{\begin{array}{r} 3700{,}36 \\ 37003{,}6 \\ 370036, \\ 3700360, \\ 37003600, \end{array}\right.$$

Q. How do you multiply a decimal number by 10, 100, 1000, &c.? If there are not as many decimal figures as there are ciphers in the multiplier, what do you do?

## DIVISION OF DECIMAL FRACTIONS.

§ **129**. Division of Decimal Fractions is similar to that of simple numbers.

We have just seen, that, if one decimal fraction be multiplied by another, the product will contain as many places of decimals as there were in both the factors

Now, if this product be divided by one of the factors the quotient will be the other factor (see § **35**.). Hence, in division, the dividend must contain just as many decimal places as the divisor and quotient together. *The quotient, therefore, will contain as many places as the dividend, less those of the divisor.*

EXAMPLES.

1. Divide 1,38483 by 60,21.

There are five decimal places in the dividend, and two in the divisor: there must therefore be three places in the quotient: hence one 0 must be prefixed to the 23, and the decimal point placed before it.

OPERATION.

```
60,21)1,38483(23
      1 2042
      ------
       18063
       18063
      ------
 Ans.  ,023.
```

Hence, for the division of decimals we have the following

RULE.

*Divide as in simple numbers, and point off in the quotient, from the right hand, so many places for decimals as the decimal places in the dividend exceed those in the divisor; and if there are not so many, supply the deficiency by prefixing ciphers.*

Q. If one decimal fraction be multiplied by another, how many decimal places will there be in the product? How does the number of decimal places in the dividend compare with those in the divisor and quotient? How do you determine the number of decimal places in the quotient? If the divisor contains four places and the dividend six, how many in the quotient? If the divisor contains three places and the dividend five, how many in the quotient? Give the rule for the division of decimals.

EXAMPLES.

1. Divide 2,3421 by 2,11. *Ans.* 1,11.
2. Divide 12,82561 by 3,01. *Ans.* 4,261.
3. Divide 33,66431 by 1,01. *Ans.* 33,331.
4. Divide ,010001 by ,01. *Ans.* 1,0001.
5. Divide 8,2470 by ,002. *Ans.* 4123,5.
6. What is the quotient of 37,57602, divided by 3? By ,3? By ,03? By ,003? By ,0003?

7. What is the quotient of 129,75896, divided by 8? By ,08? By ,008? By ,0008? By ,00008?

8. What is the quotient of 187,29900, divided by 9? By ,9? By ,09? By ,009? By ,0009? By ,00009?

9. What is the quotient of 764,2043244, divided by 6? By ,06? By ,006? By ,0006? By ,00006? By ,000006?

§ **130**. NOTE 1. When any decimal number is to be divided by 10, 100, 1000, &c. the division is made by removing the decimal point as many places to the left as there are 0's in the divisor; and if there be not so many figures on the left of the decimal point, the deficiency must be supplied by prefixing ciphers.

$$27{,}69 \text{ divided by} \begin{cases} 10 \\ 100 \\ 1000 \\ 10000 \end{cases} = \begin{cases} 2{,}769 \\ {,}2769 \\ {,}02769 \\ {,}002769 \end{cases}$$

$$642{,}89 \text{ divided by} \begin{cases} 10 \\ 100 \\ 1000 \\ 10000 \\ 100000 \end{cases} = \begin{cases} 64{,}289 \\ 6{,}4289 \\ {,}64289 \\ {,}064289 \\ {,}0064289 \end{cases}$$

Q. How do you divide a decimal number by 10, 100, 1000, &c.? If there be not as many figures to the left of the decimal point as there are ciphers in the divisor, what do you do?

§ **131**. NOTE 2. When there are more decimal places in the divisor than in the dividend, annex as many ciphers to the dividend as are necessary to make its decimal places equal to those of the divisor; *all the figures of the quotient will then be whole numbers.*

EXAMPLES.

1. Divide 4397,4 by 3,49.

We annex one 0 to the dividend. Had it contained no decimal place we should have annexed two.

OPERATION.

```
3,49)4397,40(1260
     349
     ---
      907
      698
      ----
      2094
      2094
      ----
   Ans. 1260.
```

2. Divide 2194,02194 by ,100001. *Ans.* 21940
3. Divide 9811,0047 by ,325947. *Ans.* 30100.
4. Divide ,1 by ,0001. *Ans.* 1000.
5. Divide 10 by ,1. *Ans.* 100.
6. Divide 6 by ,6? By ,06? By ,006? By ,2? By ,3? By ,003? By ,5? By ,05? By ,005? By ,000012?

Q. If there are more decimal places in the divisor than in the dividend, what do you do? What will the figures of the quotient then be?

§ **132**. Note 3. When it is necessary to continue the division farther than the figures of the dividend will allow, we may annex ciphers and consider them as decimal places of the dividend.

EXAMPLES.

1. Divide 4,25 by 1,25.

In this example we annex one 0 and then the decimal places in the dividend will exceed those in the divisor by 1.

OPERATION.

```
1,25)4,25(3,4
     3,75
     ----
      500
      500
      ---
Ans. 3,4.
```

2. Divide ,2 by ,06.

We see in this example that the division will never terminate. In such cases the division should be carried to the third or fourth place, which will give the answer true enough for all practical purposes, and the sign + should then be written, to show that the division may be still continued.

OPERATION.

```
,06),20(3,33+
     18
     --
      20
      18
      --
       20
       18
       --
        20
Ans. 3,33+.
```

3. Divide 37,4 by 4,5. *Ans.* 8,3111+
4. Divide 586,4 by 375. *Ans.* 1,563+.
5. Divide 94,0369 by 81,032. *Ans.* 1,160+.

Q. How do you continue the division after you have brought down all the figures of the dividend? What sign do you place after the quotient? What does it show?

## APPLICATIONS IN THE FOUR PRECEDING RULES.

1. A merchant sold 4 parcels of cloth, the first contained 127 and 3 thousandths yards; the 2nd, 6 and 3 tenths yards; the 3rd, 4 and one hundredths yards; the 4th, 90 and one millionth yards: how many yards did he sell in all? *Ans.* 227,313001*yd.*

2. A merchant buys three chests of tea, the first contains 60 and one thousandth *lb.*; the second, 39 and one ten thousandth *lb.*; the third, 26 and one-tenth *lb.*: how much did he buy in all? *Ans.* 125,1011*lb.*

3. What is the sum of $20 and three hundredths; $4 and one-tenth, $6 and one thousandth, and $18 and one hundredth? *Ans.* $48,141.

4. A puts in trade $504,342; B puts in $350,1965; C puts in $100,11; D puts in $99,334; and E puts in $9001,32: what is the whole amount put in? *Ans.* $10055,3025.

5. B has $936, and A has $1, 3 dimes and 1 mill: how much more money has B than A? *Ans.* $934,699.

6. A merchant buys 37,5 yards of cloth, at one dollar twenty-five cents per yard: how much does the whole come to? *Ans.* $46,875.

7. A farmer sells to a merchant 13,12 cords of wood at $4,25 per cord, and 13 bushels of wheat at $1,06 per bushel: he is to take in payment 13 yards of broadcloth at $4,07 per yard, and the remainder in cash: how much money did he receive? *Ans.* $16,63.

8. If 12 men had each $339 one dime 9 cents and 3 mills, what would be the total amount of their money? *Ans.* $4070,316.

9. If one man can remove 5,91 cubic yards of earth in a day, how much could nineteen men remove? *Ans.* 112,29*yd.*

10. What is the cost of 8,3 yards of cloth at $5,47 per yard? *Ans.* $45,401.

11. If a man earns one dollar and one mill per day, how much will he earn in a year? *Ans.* $365,365.

12. What will be the cost of 375 thousandths of a cord of wood, at $2 per cord? *Ans.* $0,75.

13. A man leaves an estate of \$1473,194 to be equally divided among 12 heirs: what is each one's portion?

*Ans.* \$122,766$\frac{1}{6}$

## REDUCTION OF VULGAR FRACTIONS TO DECIMALS.

§ **133**. The value of every vulgar fraction is equal to the quotient arising from dividing the numerator by the denominator (see § **44**.)

### EXAMPLES.

1. What is the value in decimals of $\frac{9}{2}$.

We first divide 9 by 2 which gives a quotient 4, and 1 for a remainder. Now 1 is equal to 10 tenths. If then we add a cipher, 2 will divide 10, giving the quotient 5 tenths. Hence the true quotient is 4,5.

OPERATION.

$\frac{9}{2}=4\frac{1}{2}$, but,

$4\frac{1}{2}=4\frac{10}{2}=4,5$

2. What is the value of $\frac{13}{4}$.

We first divide by 4 which gives a quotient 3 and a remainder 1. But 1 is equal to 100 hundredths. If then we add two ciphers, 4 will divide the 100, giving a quotient of 25 hundredths.

OPERATION.

$\frac{13}{4}=3\frac{1}{4}$; but

$3\frac{1}{4}=3\frac{100}{4}=3,25.$

Hence, to reduce a vulgar fraction to a decimal, we have the following

### RULE.

I. *Annex one or more ciphers to the numerator and then divide by the denominator.*

II. *If there is a remainder, annex a cipher or ciphers, and divide again, and continue to annex ciphers and to divide until there is no remainder or until the quotient is sufficiently exact: the number of decimal places to be pointed off in the quotient is the same as the number of ciphers used; and when there are not so many, ciphers must be prefixed.*

Q. What is the value of a fraction equal to? What is the value of four-halves? What is the decimal value of one-half? Of three-halves? Of six-fourths? Of nine-halves? Of seven-halves? Of five-fourths? Of one-fourth? Give the rule for reducing a vulgar fraction to a decimal.

EXAMPLES.

1. Reduce $\frac{635}{125}$ to its equivalent decimal.

We here use two ciphers and therefore point off two decimal places in the quotient.

OPERATION.

```
125)635(5,08
    625
    ----
     1000
     1000
     ----
```

2. Reduce $\frac{1}{4}$ and $\frac{9}{1129}$ to decimals.
*Ans.* ,25 and ,00797+.

3. Reduce $\frac{12}{480}$, $\frac{27}{39}$, $\frac{3}{1000}$, and $\frac{11}{60006}$ to decimals.
*Ans.* ,025; ,692+; ,003; ,000183+.

4. Reduce $\frac{1}{2}$ and $\frac{5}{1785}$ to decimals. *Ans.* ,5 and ,0028+.

5. Reduce $\frac{3149571123}{2104568891}$ to a decimal. *Ans.* 1,496+.

6. Reduce $\frac{8}{6}$, $\frac{1375}{8436}$, $\frac{3265}{4121}$, $\frac{574}{123}$ to decimals.
*Ans.* 1,333 +; 0,162 +; 0,792+; 4,666+.

## REDUCTION OF DENOMINATE DECIMALS.

§ **134**. We have seen that a denominate number is one in which the *kind* of unit is denominated or expressed (see § **45**).

A denominate decimal is a decimal fraction in which the kind of unit that has been divided is expressed. Thus, ,5 of a £, and ,6 of a shilling, are denominate decimals. The unit that was divided in the first fraction being £1, and that in the second 1 shilling.

**Q. What is a denominate number? What is a denominate decimal? In the decimal five-tenths of a £, what is the unit? In the decimal six-tenths of a shilling, what is the unit?**

### CASE I.

§ **135**. To find the value of a denominate number in decimals of a higher denomination.

EXAMPLES.

1. Reduce 9*d* to the decimal of a £.

We first find that there are 240 pence in £1. We then divide 9*d* by 240, which gives the quotient ,0375 of a £. This is the true value of 9*d* in the decimal of a £.

OPERATION.

240*d*=£1

240)9(,0375

*Ans.* £,0375.

Hence, we have the following

**RULE.**

I. *Consider how many units of the given denomination make one unit of the denomination to which you would reduce.*

II. *Divide the given denominate number by the number so found, and the quotient will be the value in the required denomination.*

Q. How do you find the value of a denominate number in a decimal of a higher denomination?

2. Reduce 7 drams to the decimal of a *lb.* avoirdupois. *Ans.* ,02734375*lb.*
3. Reduce 26*d* to the decimal of a £. *Ans.* ,1083333+.
4. Reduce ,056 poles to the decimal of an acre. *Ans.* ,00035A.
5. Reduce 14 minutes to the decimal of a day. *Ans.* ,0097222*da.*+.
6. Reduce ,21 pints to the decimal of a peck. *Ans.* ,013125*pk.*
7. Reduce 3 hours to the decimal of a day. *Ans.* ,125.
8. Reduce 375678 feet to the decimal of a mile. *Ans.* 71,151+.
9. Reduce 36 yards to the decimal of a rod.
10. Reduce ,5 quarts to the decimal of a barrel.

**CASE II.**

§ **136.** To reduce denominate numbers of different denominations to an equivalent decimal of a given denomination.

**EXAMPLES.**

1. Reduce £1 4*s* $9\frac{3}{4}d$ to the denomination of pounds.

We first reduce 3 farthings to the decimal of a penny, by dividing by 4. We then annex the quotient ,75 to the 9 pence. We next divide by 12 giving ,8125 which is the decimal of a shilling. This we annex to the pounds and then divide by 20.

OPERATION.

$\frac{3}{4}d = ,75d$, hence,

$9\frac{3}{4}d = ,975d$,

12)9,75*d*

,8125*s*, and

20)4,8125*s*

£,240625, therefore,

£1 4*s* $9\frac{3}{4}d$ = £1,240625.

Hence, we have the following

### RULE.

*Divide the lowest denomination named, by that number which makes one of the denomination next higher, annexing ciphers if necessary: then annex this quotient to the next higher denomination, and divide as before: proceed in the same manner through all the denominations to the last: the last result will be the answer sought.*

2. Reduce £19 17*s* 3¼*d* to the decimal of a £.
*Ans.* £19,863+.
3. Reduce 15*s* 6*d* to the decimal of a £. *Ans.* £,775.
4. Reduce 7½*d* to the denomination of shillings.
*Ans.* ,625*s*.
5. Reduce 2*lb.* 5*oz.* 12*pwt.* 16*gr.*, Troy, to the decimal of a *lb.* *Ans.* 2,469444*lb.*+.
6. Reduce 3 feet 9 inches to the denomination of yards.
*Ans.* 1,25*yd.*
7. Reduce 1*lb.* 12*dr.*, avoirdupois, to the denomination of pounds. *Ans.* 1,046875*lb.*
8. Reduce 5 leagues 2 furlongs to the denomination of leagues. *Ans.* 5,0833+

Q. How do you reduce denominate numbers of different denominations, to equivalent decimals of a given denomination?

### CASE III.

§ **137**. To find the value of a denominate decimal in terms of integers of inferior denominations.

#### EXAMPLES.

1. What is the value of ,832296 of a £.

We first multiply the decimal by 20 which brings it to shillings, and after cutting off from the right as many places for decimals as in the given number, we have 16*s* and the decimal ,645920 over. This we reduce to pence by multiplying by 12, and then reduce to farthings by multiplying by 4.

OPERATION.

```
      ,832296
           20
    16,645920
           12
     7,751040
            4
     3,004160
Ans. 16s 7d 3far.
```

Hence, the following

**RULE.**

I. *Consider how many in the next less denomination make one of the given denomination, and multiply the decimal by this number. Then cut off from the right hand as many places as there are in the given decimal*

II. *Multiply the figures so cut off by the number which it takes in the next less denomination to make one of a higher, and cut off as before. Proceed in the same way to the lowest denomination: the figures to the left will form the answer sought.*

2. What is the value of ,002084*lb*. Troy? *Ans*. 12,00384*gr*.
3. What is the value of ,625 of a *cwt*.? *Ans*. 2*qr*. 14*lb*.
4. What is the value of ,625 of a gallon? *Ans*. 2*qt*. 1*pt*.
5. What is the value of £,3375? *Ans*. 6*s* 9*d*.
6. What is the value of ,3375 of a ton? *Ans*. 6*cwt*. 3*qr*.
7. What is the value of ,05 of an acre? *Ans*. 8*P*.
8. What is the value of ,875 pipes of wine? *Ans*. 1*hhd*. 47*gal*. 1*qt*.
9. What is the value of ,125 hogsheads of beer? (see § **67**.) *Ans*. 6*gal*. 3*qt*.
10. What is the value of ,375 of a year of 365 days? *Ans*. 136*da*. 21*hr*.
11. What is the value of ,085 of a £? *Ans*. 1*s* 8¼*d*+.
12. What is the value of ,86 of a *cwt*.? *Ans*. 3*qr*. 12*lb*. 5*oz*. 1,92*dr*.
13. What is the difference between ,82 of a day and ,32 of an hour? *Ans*. 19*hr*. 21*m*. 36*sec*.
14. What is the value of 1,089 miles? *Ans*. 1*m* 28*rd*. 7*ft*. 11,04*in*.
15. What is the value of ,09375 of a pound avoirdupois weight? *Ans*. 1*oz*. 8*dr*.
16. What is the value of ,28493 of a year of 365 days? *Ans*. 103*da*. 23*hr*. 59*m*. 12,48*sec*.
17. What is the value of £1,046? *Ans*. £1 11*d*+.
18. What is the value of £1,88? *Ans*. £1 17*s* 7*d*+.

Q. How do you find the value of a denominate decimal in integers of inferior denominations? What is the value in shillings of one-half of a £? In pence of one-half of a shilling?

# RULE OF THREE.

§ **138.** If 1 yard of cloth cost $2, how much will 6 yards cost at the same rate?

It is plain that 6 yards of cloth, at the same rate will cost 6 times as much as 1 yard, and therefore the whole cost is found by multiplying $2 by 6, giving $12 for the cost. In this example there are four numbers considered, viz, 1 yard of cloth, 6 yards of cloth, $2 and $12: these numbers are called *terms*. Three of these terms were known or given in the question, and the other was to be found.

| | |
|---|---|
| 1 yard of cloth is the | 1st term; |
| 6 yards of cloth is the | 2nd term; |
| $2 is the - - - - | 3rd term; |
| $12 is the - - - - | 4th term. |

Now the 2nd term 6 contains the first term 1, 6 times, and the 4th term 12 contains the 3rd term 2, 6 times—that is, *the 2nd term is as many times greater than the 1st, as the 4th term is greater than the 3rd.*

This relation between four numbers is called *proportion*: and generally

*Four numbers are in proportion, when the 2nd term is as many times greater or less than the 1st, as the 4th term is greater or less than the 3rd.*

We express that four numbers are in proportion thus:

1 : 6 :: 2 : 12.

That is, we write the numbers in the same line and place two dots between the 1st and 2nd terms, four between the 2nd and 3rd, and two between the 3rd and 4th terms. We read the proportion thus,

as 1 is to 6, so is 2 to 12.

*The 1st and 2nd terms of a proportion always express quantities of the same kind, and so likewise do the 3rd and 4th terms.* As in the example.

| yd. | | yd. | | $ | | $ |
|---|---|---|---|---|---|---|
| 1 | : | 6 | :: | 2 | : | 12. |

Q. If 1 yard of cloth cost \$2, what will 6 yards cost? How many numbers are considered in this question? What are they called? How many were known or given? Name the terms. How many times does the 2nd term contain the first? How many times does the fourth contain the third? How many times is the second term greater than the first? When are four numbers in proportion? How are they written? How are they read? What terms of a proportion express quantities of the same kind? If 1 yard of cloth cost \$3 what will 2 yards cost at the same rate? What will 3 cost? 4? 5? 6? 7? 8? 9? 10? If 1 yard of cloth cost \$4 what will 2 yards cost? What will 3 yards cost? 4? 5? 6? 7? 8? 9? 10?

§ **139**. The numbers

$$2 : 4 :: 8 : 16$$

are in proportion since the 2nd term is two times greater than the 1st, and the 4th term two times greater than the 3rd. And when four numbers are in proportion, the quotient of the 2nd term divided by the 1st, is equal to the quotient of the 4th term divided by the 3rd. This quotient is called the *ratio* of the proportion. Thus 2 is the ratio of the proportion

$$2 : 4 :: 8 : 16;$$

The ratio of *two* numbers, simply expresses how many times the second number contains the first. Hence, it is equal to the quotient of the second number divided by the first.

Thus, the ratio of 3 to 9 is 3, since $9 \div 3 = 3$. The ratio of 2 to 4 is 2, since $4 \div 2 = 2$.

We may also compare a larger number with a less. For example, the ratio of 4 to 2 is $\frac{1}{2}$; for, $2 \div 4 = \frac{1}{2}$. The ratio of 9 to 3 is $\frac{1}{3}$, since $3 \div 9 = \frac{1}{3}$.

In every proportion, the ratio of the 1st term to the 2nd, is equal to the ratio of the 3rd term to the 4th.

EXAMPLES.

1. What is the ratio of 9 to 18? *Ans*. 2.
2. What is the ratio of 6 to 24? *Ans*. 4.
3. What is the ratio of 12 to 48? *Ans*. 4.
4. What is the ratio of 11 to 13? *Ans*. $1\frac{2}{11}$.
5. What part of 20 is 4? or what is the ratio of 20 to 4? *Ans*. $\frac{1}{5}$.

6. What part of 100 is 30? or what is the ratio of 100 to 30? *Ans.* $\frac{3}{10}$.

7. What are the ratios of the proportions

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 3 | : | 9 | :: | 12 | : | 36. | *Ans.* 3. |
| 2 | : | 10 | :: | 12 | : | 60. | *Ans.* 5. |
| 4 | : | 2 | :: | 8 | : | 4. | *Ans.* $\frac{1}{2}$. |
| 9 | : | 1 | :: | 90 | : | 10. | *Ans.* $\frac{1}{9}$. |
| 16 | : | 15 | :: | 48 | : | 45. | *Ans.* $\frac{15}{16}$. |

Q. When four numbers are in proportion, what is the second term divided by the first, equal to? What is this quotient called? What does the ratio of two numbers express? What is it equal to? What is the ratio of 1 to 5? Of 2 to 8? Of 3 to 27? Of 6 to 36? Of 12 to 144? Of 9 to 81? Of 10 to 100? Of 10 to 1? Of 12 to 2? Of 8 to 2? Of 8 to 1? In every proportion, what is the ratio of the 1st term to the 2nd equal to?

§ **140.** *Ex.* 2. If 4*lb.* of tea cost $8, what will 12*lb.* cost at the same rate?

```
   lb.  lb.    $    $
As 4 : 12 :: 8 : Ans.
             12
           4)96
            $24 the cost of 12lb. of tea.
```

Or,

$$\frac{12}{4}\times8=3\times8=24.$$

*Ans.* $24.

It is evident that the 4th term, or cost of 12*lb.* of tea, must be as many times greater than $8 as 12*lb.* is greater than 4*lb.* But since the quotient of 12 divided by 4 expresses how many times 12 is greater than 4, it follows that the fourth term will be equal to $8 multiplied by this quotient: that is, equal to $8 multiplied by 3, or equal to $24. But we obtain the same result whether we multiply the 3rd term $8 by the quotient 3, or first multiply it by the 2nd term and then divide the product by the 1st term; and the same may be shown for every proportion.

Hence we conclude,

*That the 4th term of every proportion may be found by multiplying the 2nd and 3rd terms together, and dividing their product by the 1st term.*

Q. How do you find the fourth term of a proportion, when the first three terms are known?

EXAMPLES.

1. The first three terms of a proportion are 1, 2, and 3: what is the 4th? *Ans.* 6

2. The first three terms are 6, 2, and 1: what is the 4th? *Ans.* $\frac{1}{3}$.

3. The first three terms are 10, 3, and 1: what is the 4th? *Ans.* $\frac{3}{10}$.

§ **141**. *The 1st and 4th terms of a proportion are called the two extremes, and the 2nd and 3rd terms are called the two means.*

Now, since the 4th term is obtained by dividing the product of the 2nd and 3rd terms by the 1st term, and since the product of the divisor by the quotient is equal to the dividend, it follows,

*That in every proportion the product of the two extremes is equal to the product of the two means.*

Thus in the first example,

1 : 6 :: 2 : 12 we have, $1\times12=6\times2=12$ and in the proportion, 4 : 12 :: 8 : 24 $4\times24=8\times12=96$.

Q. What is the product of the extremes equal to? If the product of the extremes be divided by one of them what will the quotient be? If it be divided by one of the means, what will the quotient be?

§ **142**. The Rule of Three takes its name from the circumstance that three numbers are always given to find a fourth, which shall bear the same proportion to one of the given numbers as exists between the other two. We have, for finding the 4th term, the following

GENERAL RULE.

I. *Reduce the two numbers which have different names from the answer sought, to the lowest denomination named in either of them.*

II. *Set the number which is of the same kind with the answer sought in the third place, and then consider from the nature of the question whether the answer will be greater or less than the third term.*

III. *When the answer is greater than the third term, write the least of the remaining numbers in the first place, but when it is less, place the greater there.*

IV. *Then multiply the second and third terms together and* *divide the product by the first term: the quotient will be the fourth term or answer sought, and will be of the same denomination as the third term.*

*Ex.* 3. If 48 yards of cloth cost $67,25, what will 144 yards cost at the same rate?

In this example, as the answer is to be dollars, we place the $67,25 in the 3rd term. Then, as 144 yards of cloth will cost more than 48 yards, the 4th term must be greater than the 3rd, and therefore, we write the least of the two remaining numbers in the first place. The product of the 2nd and 3rd terms is $9684,00: then dividing by the 1st term we obtain $201,75 for the cost of 144 yards of cloth.

OPERATION.

```
yd.   yd.       $        $.
48 : 144 : : 67,25 : Ans.
                144
              26900
             26900
             6725
        48)9684,00($201,75
           96
             84
             48
             360
             336
              240
              240
```

**Q.** From what does the Rule of Three take its name? What is the first thing to be done in stating a question? Which number do you make the third term? How do you determine which to put in the first? After stating the question, how do you find the 4th term? What will be its denomination?

*Ex.* 4. If 6 men can dig a certain ditch in 40 days, how many days would 30 men be employed in digging it?

As the answer must be days, the 40 days are written in the third place. Then as it is evident that 30 men will do the same work in a shorter time than 6 men, it is plain that the fourth term must be less than the third: therefore, 30 men, the greater of the remaining numbers, is written in the first term. Besides, it is plain that the fourth term must be just so many times less than 40, as 6 is less than 30.

OPERATION.

```
men  men   days   days.
30 :  6 :: 40  :  Ans.
               6
         3|0)24|0
     Ans.   8 days.
```

*Ex.* 5. If 25 yards of cloth cost £2 3*s* 4*d*, what will 5 yards cost at the same rate?

When we come to divide the product of the 2nd and 3rd terms by the first, it is found the £10 does not contain 25. We then reduce to the next lower denomination and divide as in division of denominate numbers.

OPERATION.

```
yd.   yd.     £   s   d.
25  :  5  ::  2   3   4  :  Ans.
                      5
        25)£10  16s  8d
            20
        25)216(8s
           200
            16
            12
        25)200(8d
           200
```

*Ex.* 6. If 3*cwt.* of sugar cost £9 2*s* 0*d*, what will 4*cwt.* 3*qr.* 26*lb.* cost at the same rate?

```
              3cwt.     4cwt. 3qr. 26lb.
               4         4                   £9  2s 0d
              12        19                    20
            {  7         7                   182s
4×7=28      { 84       133                    12
            {  4         4                  2184
            336lb.  :  558lb.  ::  2184d  :  Ans.
                                    558
                                  17472
                                 10920
                                10920
                          336)1218672(3627d
                              1008
                               2106         12)3627
                               2016         20)30s..3d
                                907          £15...2s
                                672
                               2352
                               2352
                                      Ans. £15 2s 3d.
```

We first reduce the 1st and 2nd terms to pounds, then the 3rd term to pence. The answer comes out in pence, and is afterwards reduced to pounds shillings and pence.

### PROOF.

**§ 143.** The product of the two means is equal to the product of the extremes (see **§ 141**). Hence, if either of these equal products be divided by one of the mean terms the quotient will be the other. Therefore,

*Divide the product of the extremes by one of the mean terms, and if the work is right the quotient will be the other mean term.*

### EXAMPLES.

1. The 1st term is 4, the 2nd 8, the 3rd 12, and the answer 24: is the answer true?

The product of the extremes is 96. If this be divided by 8 the quotient is 12; if by 12 the quotient is 8: hence, the answer was true.

OPERATION OF PROOF.

$24 \times 4 = 96$

8)96(12; or

12)96(8

### APPLICATIONS.

1. If 4 hats cost $12, what will 55 cost at the same rate? *Ans.* $165.

2. What is the value of 2*cwt.* of sugar at 5*d* per pound? *Ans.* £4 13*s* 4*d*.

3. If 40 yards of cloth cost $170, what will 325 yards cost? *Ans.* $1381,25.

4. If 240 sheep yield 660*lb.* of wool, how many pounds will be obtained from 1200? *Ans.* 3300*lb.*

5. If 2 gallons of molasses cost 65 cents, what will 3 hogsheads cost? *Ans.* $61,42½.

6. If a man travels at the rate of 210 miles in 6 days, how far will he travel in a year supposing him not to travel on Sundays? *Ans.* 10955 *miles.*

7. If 1 yard of cloth cost $3,25, what will be the cost of 3 pieces each containing 25 yards? *Ans.* $243,75.

8. If 30 barrels of flour will support 100 men for 40 days, how long would it subsist 25 men? *Ans.* 160 days.

9. If 30 barrels of flour will support 100 men for 40 days, how long would it subsist 400 men? *Ans.* 10 *days.*

10. A owes B £679 6*s*, but compounds with him by paying 3*s* 4*d* on the pound: how much does B receive of his debt? *Ans.* £113 4*s* 4*d*.

11. If 90 bushels of oats will feed 40 horses for 6 days, how long would 450 bushels last them? *Ans.* 30 *days*.

12. If 5*cwt.* 3*qr.* 14*lb.* of sugar cost £6 1*s* 8*d*, what will 35*cwt.* 28*lb.* cost? *Ans.* £36 10*s*.

13. What is the cost of 3*cwt.* of coffee at 15*d* per pound? *Ans.* £21.

14. If 3 quarters of a yard of velvet cost 7*s* 3*d*, how many yards can be bought for £13 15*s* 6*d*?

*Ans.* 28*yd.* 2*qr.*

15. If an ingot of gold weighing 9*lb.* 9*oz.* 12*pwt.* be worth £470 8*s*, what is that per grain? *Ans.* 2*d*.

16. Bought 4 bales of cloth, each containing 6 pieces, and each piece 27 yards, at £16 4*s* per piece; what is the value of the whole, and the cost per yard?

*Ans.* £388 16*s* at 12*s* per yard.

17. What will be the cost of 72 yards of cloth, at the rate of £5 12*s* for 9 yards? *Ans.* £44 16*s*.

18. A person's annual income is £146: how much is that per day? *Ans.* 8*s*.

19. If 3 paces or common steps of a person, be equal to 2 yards, how many yards will 160 paces make?

*Ans.* 106*yd.* 2*ft.*

20. What length must be cut off from a board that is 9 inches wide, to make a square foot, that is, as much as is contained in 12 inches in length and 12 in breadth?

*Ans.* 16 *inches*.

21. If 750 men require 22500 rations of bread for a month, how many rations will a garrison of 1200 men require? *Ans.* 36000.

22. If 7*cwt.* 1*qr.* of sugar cost $64,96, what will be the price of 4*cwt.* 2*qr.*? *Ans.* $40,32.

23. The clothing of a regiment of foot of 750 men amounts to £2831 5*s*: what will it cost to clothe a body of 3500 men? *Ans.* £13212 10*s*.

24. How many yards of carpeting, that is 3 feet wide, will cover a floor that is 27 feet long and 20 feet broad?

*Ans.* 60 *yards*.

25. What is the cost of 6 bushels of coal at the rate of £1 14*s* 6*d* the chaldron? *Ans.* 5*s* 9*d*.

26. If 6352 stones of 3 feet long will complete a certain quantity of wall, how many stones of 2 feet long will raise the like quantity? *Ans.* 9528.

27. If a person can count 300 in two minutes, how many can he count in a day? *Ans.* 216000.

28. A garrison of 536 men have provisions for 365 days: how long will those provisions last if the garrison be increased to 1124 men? *Ans.* 174 days and $\frac{64}{1124}$.

29. What will be the tax upon £763 15*s* at the rate of 3*s* 6*d* per pound sterling? *Ans.* £133 13*s* 1½*d*.

30. What will be the tax on $3758, at the rate of 4 mills on the dollar? *Ans.* $15,032.

31. A certain work can be raised in 12 days by working 4 hours each day: how long would it require to raise the work by working 6 hours per day? *Ans.* 8 *days*.

32. What quantity of corn can I buy for 90 guineas, at the rate of 6 shillings a bushel? *Ans.* 315 bushels.

33. A person failing in trade owes £977, at which time he has in money, goods, and recoverable debts £420 6*s* 3¼*d*: now supposing an equal division among his creditors, how much will they get on the pound?

*Ans.* 8*s* 7¼*d*.

34. A pasture of a certain extent having supplied a body of horse, consisting of 3000, with forage for 18 days, how many days would the same pasture have supplied a body of 2000 horse? *Ans.* 27 *days*.

35. Suppose a gentleman's income to be 600 guineas a year, and that he spends 25*s* 6*d* per day, one day with another: how much will he have at the end of the year?

*Ans.* £164 12*s* 6*d*.

36. What is the cost of 30 pieces of lead, each weighing 1*cwt.* 12*lb.* at the rate of 16*s* 4*d* the *cwt.*?

*Ans.* £27 2*s* 6*d*.

37. The governor of a besieged place has provisions for 54 days at the rate of 2*lb.* of bread per ration, but is desirous to prolong the siege to 80 days, in expectation of succor: in that case what must be the ration of bread?

*Ans.* $1\frac{28}{80}$*lb*

38. If a person pays half a guinea a week for his board, how long can he be boarded for £21? *Ans.* 40 weeks.

39. What is the value of a year's rent of 547 acres of land at the rate of 15*s* 6*d* the acre? *Ans.* £423 18*s* 6*d*.

40. If a person drinks 20 bottles of wine per month, when it costs 2*s* per bottle, how much can he drink without increasing the expense when it costs 2*s* 6*d* per bottle? *Ans.* 16 *bottles*.

41. A merchant bought 21 pieces of cloth, each containing 40 yards, for which he paid $1260; he sold the cloth at $1,75 per yard: did he make or lose by the bargain? *Ans.* he gained $210.

42. A cistern containing 200 gallons is filled by a pipe which discharges 3 gallons in 5 minutes: but the cistern has a leak which empties 1 gallon in 5 minutes. Now if the water begins to run in, when the cistern is empty, how long will it be in filling? *Ans.* 8 *hours* 20 *minutes*.

43. What will be the cost of 895 feet of timber, at $6 per hundred feet?

In this example, 100 feet of timber, is to the given quantity 895 feet, as $6, the cost of 100 feet, is to $53,70, the cost of 895 feet. If the timber had been sold at the rate of $6 per thousand feet, the cost of 895 feet would have been $5,37, for we should have divided by 1000, instead of 100; that is, we should have removed the separating point in the product three places to the left. Hence, to find the cost of things sold by the 100, or 1000, we have the following

OPERATION.

```
100 : 895 :: 6 : Ans.
        6
100)5370
   53,70
```

*Ans.* $53,70.

RULE.

*Multiply the number of things by the price, and if the things be reckoned by the* 100, *cut off two places from the right, and if reckoned by the* 1000, *cut off three, and the figures to the left will be the answer in the same denomination as the given price.*

44. What will be the cost of 1350 feet of boards at $11 per hundred? *Ans.* $148,50.

45. What will be the cost of 36578 bricks at $6,50 per thousand? *Ans.* $237,75+.

46. What will be the cost of 6359 feet of boards at $9,25 per 100 feet? *Ans.* $588,20+.

47. What will be the cost of 13918 feet of timber at $14,37 per thousand? *Ans.* $200,00+.

48. What will 18759 oranges cost at $5,50 per hundred? *Ans.* $1031,74+.

49. What is the cost of 6559 feet of round timber at $9,25 per 100 feet? *Ans.* $606,70+.

50. What is the cost of 37032 feet of square timber at $85,72 per thousand feet? *Ans.* $3174,38+.

**Q.** How do you find the cost of things sold by the hundred? How do you find the cost of things sold by the thousand?

## QUESTIONS INVOLVING FRACTIONS.

### EXAMPLES.

1. If $\frac{3}{8}$ of a yard of cloth cost $3,20, what will $2\frac{1}{2}$ yards cost?

We state the question exactly as in whole numbers. In multiplying the 2nd, and 3rd terms together, we observe the rules for multiplying fractions, and in dividing by the 1st term, the rules for division. Thus, in this example, we invert the terms of the divisor and multiply.

OPERATION.

$$\frac{3}{8} : 2\frac{1}{2} :: 3,20 : Ans.$$

$$\begin{array}{r} 2\frac{1}{2} \\ \hline 6,40 \\ \text{by multiplying by } \frac{1}{2} \quad 1,60 \\ \hline 8,00 \end{array}$$

$$8,00 \div \frac{3}{8} = 8,00 \times \frac{8}{3} = \frac{64,00}{3} = \$21,33+.$$

2. If $\frac{3}{4}oz.$ cost £$\frac{1}{12}$, what will 1*oz.* cost? *Ans.* £1 5*s* 8*d*.

3. If $\frac{3}{16}$ of a ship cost £273 2*s* 6*d*, what will $\frac{5}{32}$ of her cost? *Ans.* £227 12*s* 1*d*.

4. A mercer bought $3\frac{1}{2}$ pieces of silk each containing

24⅓ yards. He paid 6*s* ½*d* per yard: what does the whole come to? *Ans.* £25 14*s* 6½*d*+.

5. If 14*lb.* of sugar cost $1½, what will 6*lb.* cost?

*Ans.* $9/14.

6. If 3/5 of a yard of cloth cost 7/6 of a dollar, what will 2½ yards cost? *Ans.* $4 31/36.

7. If 2*lb.* of beef cost ⅛ of a dollar, what will 30*lb.* cost?

*Ans.* $1⅞.

8. If 14½ yards of cloth cost $19⅓, how much will 19⅞ yards cost? *Ans.* $26½.

9. If ,3 of a house cost $100,75, what would ,95 cost?

*Ans.* $319,04+.

10. A man receives 3/5 of his income and finds it equal to $3724,16: how much is his whole income?

*Ans.* $6206,93+.

## RULE OF THREE BY ANALYSIS.

§ **144**. The solution of questions in the Rule of Three by analysis consists in finding the ratio of two of the given terms, and multiplying this ratio by the other term.

The ratio of two of the terms will generally express the value or cost of a single thing.

### EXAMPLES.

1. If 3 barrels of flour cost $24, what will 7 barrels cost?

By dividing the $24 by 3 we get the the cost of 1 barrel. For, if $24 will buy 3 barrels, it is plain that ⅓ of it will buy 1 barrel. This, multiplied by 7, gives $56 the cost of 7 barrels.

OPERATION.

3)24

8

8×7=56

*Ans.* $56.

Q. In what does the solution of questions by analysis consist? What does the ratio of two of the terms express? If this ratio be multiplied by the other term what is the product? If 6 oranges cost 12 cents, how much will 8 cost? If 3 apples cost 1 cent, how much will 7 cost? What is the ratio of 3 to 1? If 9 yards of cloth cost $27, what will 15 yards cost? If 15 bushels of wheat cost $30, what will 50 bushels cost?

2. If in 29 days a man travels 58 miles, how far will he travel in 30 days? *Ans.* 60.

3. If 6 men consume 1 barrel of flour in 30 days, how much would 48 men consume?

It is evident that $\frac{1}{6}$ of a barrel would be the amount consumed by 1 man; hence, 48 times $\frac{1}{6}$ is the amount consumed by 48 men.

OPERATION.

$\frac{1}{6}\times 48=8.$

*Ans.* 8.

4. If $\frac{1}{8}$ of a barrel of flour cost $\frac{2}{5}$ of a dollar, what will $\frac{5}{16}$ cost? *Ans.* \$1.

5. If I walk 84 miles in 3 days, how far should I walk at the same rate in 9? *Ans.* 252.

6. If 8*lb.* of sugar cost \$1,28, how much will 13*lb.* cost? What is $16\times 13$? *Ans.* \$2,08.

7. If $\frac{3}{4}$ of a piece of cloth cost \$8,25, what will $\frac{9}{4}$ cost? *Ans.* \$24,75.

9. If 300 barrels of flour cost \$570, what will 200 cost? What is $\frac{2}{3}\times 570$? *Ans.* \$380.

10. If $\frac{8}{7}$ of a barrel of cider cost $\frac{9}{11}$ of a dollar, what will $\frac{3}{5}$ cost? What is $\frac{21}{40}\times\frac{9}{11}$? *Ans.* \$$\frac{189}{440}$.

## OF QUESTIONS REQUIRING TWO STATEMENTS.

§ **145**. The answer to each of the above questions has been found by a single statement. Questions, however, frequently occur in which two or more statements are necessary. In most Arithmetics, such questions are arranged under a rule called Compound Proportion, or the Double Rule of Three. They can, however, be answered by the rules already given.

### EXAMPLES.

1. If a family of 6 persons expend \$300 in 8 months, how much will serve a family of 15 persons for 20 months?

*First question.* If \$300 will support a family of 6 persons for 8 months, how many dollars will support 15 persons for for the same time?

OPERATION.

| persons. | persons. | | \$ | | \$ |
|---|---|---|---|---|---|
| 6 | : 15 | :: | 300 | : | *Ans.* |
| | | | 15 | | |
| | | | 6(4500 | | |
| | | *Ans.* | \$750 | | |

*Second question.* If $750 will support a family of 15 persons for 8 months, how much will serve them for 20 months?

OPERATION.

| *months.* | | *months.* | | $ | | $ |
|---|---|---|---|---|---|---|
| 8 | : | 20 | : : | 750 | : | *Ans.* |

$$\begin{array}{r} 750 \\ 20 \\ \hline 8)15000 \\ \hline \textit{Ans}\ \$1875. \end{array}$$

2. If 16 men build 18 feet of wall in 12 days, how many men must be employed to build 72 feet in 8 days, working at the same rate?

The first question is, how long would it take the 16 men to build the 72 feet of wall.

It is evident that 18 feet of wall, is to 72 feet, as 12 days, the time necessary to build 18 feet, is to 48 days, the time necessary to build 72 feet.

OPERATION.

| *feet.* | | *feet.* | | *days.* | | *days.* |
|---|---|---|---|---|---|---|
| 18 | : | 72 | : : | 12 | : | *Ans.* |

$$\begin{array}{r} 12 \\ 72 \\ \hline 24 \\ 84 \\ \hline 18)864(48 \text{ days.} \\ 72 \\ \hline 144 \\ 144 \\ \hline \end{array}$$

The second question is, if 16 men can build 72 feet of wall in 48 days, how many men are necessary to build it in 8 days?

Make 16 men the third term. Then as the same work is to be done in less time, more men will be necessary; therefore, the fourth term will be greater than the third, and hence 8 days are placed in the first term (see § **142**).

OPERATION.

| *days.* | | *days.* | | *men.* | | *men.* |
|---|---|---|---|---|---|---|
| 8 | : | 48 | : : | 16 | : | *Ans.* |

$$\begin{array}{r} 16 \\ 48 \\ \hline 128 \\ 64 \\ \hline 8)768( \\ \hline \textit{Ans.}\ 96\ \textit{men.} \end{array}$$

3. If a man travel 217 miles in 7 days, travelling 6 hours a day, how far would he travel in 9 days, if he travelled 11 hours a day.

| OPERATION. 1st. | | | | | OPERATION. 2nd. | | | |
|---|---|---|---|---|---|---|---|---|
| *days.* | *days.* | | *miles.* | *miles.* | *hours.* | *hours.* | *miles.* | *miles.* |
| 7 | : 9 | :: | 217 | : 279 | 6 | : 11 | :: 279 | : $511\frac{3}{6}$ |
| | | | 9 | | | | 11 | |
| | | | 7(1953 | | | | 6)3069 | |
| | | | 279 | | | | $511\frac{3}{6}$ | |
| | | | | | | | *Ans.* $511\frac{3}{6}$ *miles.* | |

4. If a pasture of 16 acres will feed 6 horses for 4 months, how many acres will feed 12 horses for 9 months? *Ans.* 72.

5. If the wages of 6 men for 14 days be $84, what will be the wages of 9 men for 11 days? *Ans.* $99.

6. If 154 bushels of oats serve 14 horses for 44 days, how long would 406 bushels last 7 horses? *Ans.* 232 *days.*

7. If 25 men can earn $6250 in 2 years, how long will it take 5 men to earn $11250? *Ans.* 18 *years.*

8. If a barrel of beer last 7 persons 12 days, how much will be drank by 42 persons in a year? *Ans.* 182*bar.* 18*gal.*

9. If 9 men can cut 36 acres of grass in 4 days, how many acres will 19 men cut in 11 days? *Ans.* 209 *acres.*

10. If 25 persons consume 300 bushels of corn in 1 year, how much will 139 persons consume in 7 years at the same rate? *Ans.* 11676*bu.*

11. If 32 men build a wall 36 feet long, 8 feet high, and 4 feet wide in 4 days; in what time will 48 men build a wall 864 feet long, 6 feet high, and 3 feet wide? *Ans.* 36 *days.*

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## REDUCTION OF CURRENCIES.

§ **146**. Previous to the act of Congress which established a uniform currency throughout the United Sates, in dollars, dimes, cents, and mills, each State had its own particular currency. This circumstance has caused the dollar to be divided differently in the different States, though the real value of it is the same in all.

Thus 1 dollar is reckoned in

| | | | |
|---|---|---|---|
| England. | at 4*s* 6*d*, | called | English, or Sterling money. |
| Canada,<br>Nova Scotia. | 5*s* | „ | Canada Currency. |
| New York,<br>Ohio,<br>North Carolina. | 8*s*<br><br>10*s* | „ | New York Currency. |
| The New England States,<br>Virginia,<br>Kentucky,<br>Tennessee. | 6*s* | „ | New England Currency. |
| New Jersey,<br>Pennsylvania,<br>Delaware,<br>Maryland. | 7*s* 6*d* | „ | Pennsylvania Currency. |
| South Carolina,<br>Georgia. | 4*s* 8*d* | „ | Georgia Currency. |

Q. How was the currency of the United States established? What are its denominations? Is the value of a dollar the same in all the states? How many shillings make a dollar in English Currency? In Canada Currency? In New York Currency? In New England Currency? In Pennsylvania Currency? In Georgia Currency?

CASE I.

§ **147**. To reduce a sum in either currency to Federal Money.

The reduction of these currencies to Federal Money consists in this: *having given any sum expressed in pounds shillings and pence, to find its value in dollars cents and mills.*

Take, for example the English Currency.

$$£1=20s=240d.$$

$$\$1=4s\ 6d=54d.$$

Hence, $£1 : \$1 :: 240 : 54$,

and consequently, $£1=\$1\times\frac{240}{54}=\$1\times\frac{40}{9}$;

that is, £1 is equal to $\frac{40}{9}$ of a dollar, which being reduced to dollars and the decimal of a dollar gives

$$£1=\$4,444+.$$

If we now multiply each side of the equality

$£1=\$1\times\frac{40}{9}$ by 2, 3, 4, 5, &c, we have

$£2=\$1\times\frac{40}{9}\times 2$, $£3=\$1\times\frac{40}{9}\times 3$

$£4=\$1\times\frac{40}{9}\times 4$, $£5=\$1\times\frac{40}{90}\times 5$, and generally, any number of pounds will be brought into dollars by multiplying by $\frac{40}{9}$.

As similar reasoning may be employed for all the currencies, we have the following

### RULE.

I. *Reduce the given sum to the decimal of a pound as in* § 135.

II. *Find the value of £1 in the fraction of a dollar.*

III. *Multiply the given sum by this fraction and the product will be the value in dollars.*

### EXAMPLES.

1. Change £27 3*s* 9*d* to Federal Money.

Having reduced the pounds, shillings, and pence to the decimal of a pound, we bring the whole to dollars by multiplying by 40 and dividing the product by 9.

OPERATION.

3*s* 9*d*=£0,1875.
£27 3*s* 9*d*=£27,1875.
$27{,}1875 \times \frac{40}{9} = 120{,}83\ 3+$.
*Ans.* $120,83 3+.

2. Change £140 18*s* 9*d* to Federal Money.
*Ans.* $626,38 8+.

By an act of Congress passed in 1832, the value at which the English pound is to be received and estimated at the custom-house, was fixed at $4,80. If we assume this as the value of the English pound, we have

$$£1 : \$1 :: 480 : 100;\ \text{hence},$$
$$£1 = \$1 \times \frac{480}{100} = \$1 \times \frac{24}{5}$$

therefore, pounds will be brought to dollars by multiplying by 24 and dividing by 5.

1. Reckoning the pound at $4,80, how many dollars in £49 8*s* 6*d*?

We first bring the sum to pounds and decimals of a pound. We next multiply by the multiplier $\frac{24}{5}$: we then reduce the product to pounds, shillings, and pence.

OPERATION.

£49 8*s* 6d=£49,425.
$49{,}425 \times \frac{24}{5} = 237{,}24$.
*Ans.* $237, 24.

**Q.** In what does the reduction of a currency to Federal Money consist? What is the value of the English pound as formerly estimated? How will you reduce pounds of this value to Federal Money? Give the General Rule for reducing any currency to Federal Money. What is the custom-house value of the pound established by act of Congress in 1832? How will you reduce pounds of this value to Federal Money?

### CANADA CURRENCY.

1. Change £25 10*s* 6*d* to Federal Money.

$1=5*s*=60*d*;
£1 : $1 :: 240 : 60
£1=$1$\frac{240}{60}$=$1×4=$4.
Hence, Canada Money is reduced to Federal Money by multiplying by 4.

OPERATION.
£25 10*s* 6*d*=£25,525
25,525×4=102,10.
*Ans.* $102,10.

2. Change £69 15*s* 5*d* to Federal Money.
*Ans.* $279,08 3+.

### NEW ENGLAND CURRENCY.

1. Change £40 6*s* 6*d* to Federal Money.

$1=6*s*=72*d*;
£1 : $1 : : 240 : 72
£1=$1×$\frac{240}{72}$=$1×$\frac{10}{3}$.
Hence, New England Currency is reduced to Federal Money by multiplying by 10 and dividing by 3.

OPERATION.
£40 6*s* 6*d*=£40,325
40,325×$\frac{10}{3}$=134,416+.
*Ans.* $134,41 6+.

2. Change £125 15*s* 8*d* to Federal Money.
*Ans.* $419,27 7+.

### NEW YORK CURRENCY.

1. Change £365 10*s* 6*d* to Federal Money.

$1=8*s*=96*d*.
£1 : $1 :: 240 : 96.
£1=$1×$\frac{240}{96}$=$$\frac{10}{4}$.
Hence, New York Currency is reduced to Federal Money by multiplying by 10 and dividing the product by 4.

OPERATION.
£365 10*s* 6*d*=£365,525
365,525×$\frac{10}{4}$=913,812+
*Ans.* $913,812+.

2. Change £20 18*s* 9*d* to Federal Money.
*Ans.* $52,343+.

PENNSYLVANIA CURRENCY.

1. Change £9 8*s* 3*d* to Federal Money.

$1=7*s* 6*d*=90*d*.

£1 : $1 :: 240 : 90.

£1=$1 × $\frac{240}{90}$=$$\frac{8}{3}$

Hence, to reduce Pennsylvania Currency to Federal Money, we multiply by 8 and divide by 3.

OPERATION.

£9 8*s* 3*d*=£9,4125

9,4125 × $\frac{8}{3}$=25,10.

*Ans.* $25,10

2. Change £19 18*s* 2*d*, to Federal Money.

*Ans.* $53,088+.

GEORGIA CURRENCY.

1. Change £187 9*s* 10*d* to Federal Money.

$1=4*s* 8*d*=56*d*.

£1 : $1 :: 240 : 56.

£1=$1 × $\frac{240}{56}$=$1 × $\frac{30}{7}$.

Hence, we reduce Georgia Currency to Federal Money by multiplying by 30 and dividing by 7.

OPERATION.

£187 9*s* 10*d*=£187,49166+

187,49166 × $\frac{30}{7}$=803,535.

*Ans.* $803,535+.

2. Change £26 14*s* 6*d* to Federal Money.

*Ans.* $114,535+.

1. Reduce £112 18*s* 9*d* in each of the currencies named, to Federal Money.

£112 18*s* 9*d*=£112,9375.

| | | |
|---|---|---|
| *Ans.* | £112,9375 Sterling Money | =$501,944+. |
| | £112,9375 Canada Currency | =$451,75. |
| | £112,9375 New England - | =$376,458+. |
| | £112,9375 New York - | =$282,343+. |
| | £112,9375 Pennsylvania - | =$301,166+. |
| | £112,9375 Georgia - | =$484,017+. |

CASE II.

§ **148**. To change a sum expressed in Federal Money to any one of the above currencies.

In changing pounds to Federal Money, we found the value of a pound in the fraction of a dollar and multiplied the pounds by this fraction.

Now, to change dollars to pounds, we must find the value of a dollar in the fraction of a pound and multiply the dollars by this fraction.

For example, in English Money,

$$£1 : \$1 :: 240 : 54, \text{ and}$$

$$\$1 = £1 \times \frac{54}{240} = £1 \times \frac{9}{40} = \frac{9}{40} \text{ of } £1.$$

*Ex.* 1. Reduce $32,789 to English or Sterling Money.

We first multiply by the fraction $\frac{9}{40}$: that is, we multiply by the numerator and divide by the denominator, this brings the sum to the decimal of a pound. We then reduce the decimal of a pound to pounds shillings and pence.

OPERATION.

$$\$32,789 \times \frac{9}{40} = 7,377525$$

| |
|---:|
| £7,377525 |
| 20 |
| 7,550500 |
| 12 |
| 6,606000 |
| 4 |
| 2,424000 |

*Ans.* £7 7*s* 6*d* 2*far*.+.

§ **149**. Hence, to pass from Federal Money to pounds shillings and pence, we have the following

RULE.

I. *Find the value of a dollar in the fraction of a pound.*
II. *Multiply the given sum by this fraction.*
III. *Reduce the product to pounds shillings and penee.*

Note. The multiplier in each case, in passing from Federal Money to pounds shillings and pence, will be the fraction used in the corresponding case in passing to Federal money, with its terms inverted.

Q. How do you pass from Federal Money to either of the currencies? How does this multiplier compare with the multiplier used in passing from pounds to Federal money?

EXAMPLES.

1. Reduce $102,85 to the several currencies.

*Ans.*

| | | | | |
|---|---|---|---|---|
| $102,85=£23 | 2*s* | 9¾*d*+ | Sterling Money. |
| $102,85=£25 | 14*s* | 3*d* | Canada Currency. |
| $102,85=£30 | 17*s* | 1*d*+ | New England Cur. |
| $102,85=£41 | 2*s* | 9½*d*+ | New York Currency. |
| $102,85=£38 | 11*s* | 4½*d* | Pennsylvania Cur. |
| $102,85=£23 | 19*s* | 11½*d*+ | Georgia Currency. |

2. Reduce $250 to the several currencies.

| | | |
|---|---|---|
| *Ans.* | $250=£56 5*s* | Sterling Money. |
| | $250=£62 10*s* | Canada Currency. |
| | $250=£75 | New England Currency. |
| | $250=£100 | New York Currency. |
| | $250=£93 15*s* | Pennsylvania Currency. |
| | $250=£58 6*s* 7$\frac{3}{4}$*d*.+ | Georgia Currency. |

The following are the rates at which foreign coins are estimated at the custom-houses of the United States.

| | |
|---|---|
| English £ by act of Congress of 1832 | $4,80 |
| Livre of France | $ ,18$\frac{1}{2}$ |
| Franc do | $ ,18$\frac{3}{4}$ |
| Silver Rouble of Russia | $ ,75 |
| Florin or Guilder of the United Netherlands | $ ,40 |
| Mark Banco of Hamburg | $ ,33$\frac{1}{3}$ |
| Real of Plate of Spain | $ ,10 |
| Real of Vellon of do. | $ ,05 |
| Milrea of Portugal | $1,24 |
| Tale of China | $1,48 |
| Pagoda of India | $1,84 |
| Rupee of Bengal | $ ,50 |

## PRACTICE.

§ **150**. Practice is a short method of finding the answers to questions in the Rule of Three, when the first term is unity.

For example, if one yard of cloth cost half a dollar, what will 60 yards cost. This is a question which may be answered by the rule called Practice.

If the cloth had been $1 per yard, the cost of 60 yards would have been $60; but since it is only a part of a dollar per yard, the whole cost will be the same part of $60, that the cost of one yard is of $1; that is, $\frac{1}{2}$ of 60. Hence the cost is $\frac{1}{2}$ of $60 or $30. *Ans.* $30.

§ **151**. One number is said to be an aliquot part of another, when it forms an exact part of it: that is, when it is contained in that other an exact number of times. Hence, an aliquot part is an exact or even part.

For example, 25 cents is an aliquot part of a dollar. It is an exact fourth part, and is contained in the dollar four times. So also, 2 months, 3 months, 4 months, and 6 months, are all aliquot parts of a year.

### TABLE OF ALIQUOT PARTS.

| *Cts.* | *Parts of $1.* | *Mo.* | *Parts of a year.* | *Days.* | *Parts of 1 mo.* | *Parts of £1.* | *Parts of 1 shilling.* |
|---|---|---|---|---|---|---|---|
| 50 | $\frac{1}{2}$ | 6 | $\frac{1}{2}$ | 15 | $\frac{1}{2}$ | 10*s* $=\frac{1}{2}$ | 6 *d*$=\frac{1}{2}$ |
| $33\frac{1}{3}$ | $\frac{1}{3}$ | 4 | $\frac{1}{3}$ | 10 | $\frac{1}{3}$ | 6*s* 8*d*$=\frac{1}{3}$ | 4 *d*$=\frac{1}{3}$ |
| 25 | $\frac{1}{4}$ | 3 | $\frac{1}{4}$ | $7\frac{1}{2}$ | $\frac{1}{4}$ | 5*s* $=\frac{1}{4}$ | 3 *d*$=\frac{1}{4}$ |
| 20 | $\frac{1}{5}$ | 2 | $\frac{1}{6}$ | 6 | $\frac{1}{5}$ | 4*s* $=\frac{1}{5}$ | 2 *d*$=\frac{1}{6}$ |
| $12\frac{1}{2}$ | $\frac{1}{8}$ | 1 | $\frac{1}{12}$ | 5 | $\frac{1}{6}$ | 3*s* 4*d*$=\frac{1}{6}$ | $1\frac{1}{2}$*d*$=\frac{1}{8}$ |
| $6\frac{1}{4}$ | $\frac{1}{16}$ | | or $\frac{1}{3}$ of | 3 | $\frac{1}{10}$ | 2*s* 6*d*$=\frac{1}{8}$ | 1 *d*$=\frac{1}{12}$ |
| 5 | $\frac{1}{20}$ | | 3 *mo.* | | | 1*s* 8*d*$=\frac{1}{12}$ | |

Q. What is practice? If one yard of cloth cost $8, what will half a yard cost? What will one quarter of a yard cost? When is one number said to be an aliquot part of another? What is an aliquot part? What are the aliquot parts of a dollar expressed in the table? What the aliquot parts of a year? What the aliquot parts of a month? What the aliquot parts of a pound? What the aliquot parts of a shilling?

EXAMPLES.

1. What is the cost of 376 yards of cloth at $0,75, or $\frac{3}{4}$ of a dollar, per yard?

OPERATION.

| *cts.* | | $ | |
|---|---|---|---|
| 50 | $\frac{1}{2}$ | 376 | |
| | | 188 | cost at 50*cts.* |
| 25 | $\frac{1}{4}$ | 94 | cost at 25*cts.* |
| 75 | $\frac{3}{4}$ | $282 | cost at $\frac{3}{4}$*doll.* |

Had the cloth cost $1 per yard, the cost of the 376 yards would have been $376. Had it cost 50*cts.* per yard, the cost would have been $\frac{1}{2}$ of $376, or $188: had it been 25*cts.* per yard, the cost would have been $\frac{1}{4}$ of $376 or $94; but the price being 75*cts.* per yard, the cost is 188+94=$282.

2. What is the cost of 196 yards of cotton, at 9*d* per yard?

196*yd.* at 6*d* or $\frac{1}{2}s=98s$
196*yd.* at 3*d* or $\frac{1}{4}s=49s$

Therefore, 196*yd.* at 9*d* or $\frac{3}{4}s=147s=$ £7 7*s.* *Ans.*

3. What will $9\frac{1}{2}$ yards of cloth cost at £1 4*s* 6*d* per yard?

| | |
|---|---|
| 9 yards at £1 = | £9 |
| $\frac{1}{2}$ yard at £1 = | 10*s* |
| 9 yards at 4*s* = | £1 16*s* |
| $\frac{1}{2}$ yard at 4*s* = | 2*s* |
| 9 yards at 6*d* = | 4*s* 6*d* |
| $\frac{1}{2}$ yard at 6*d* = | 3*d* |
| Total cost | £11 12*s* 9*d*. |

4. What is the cost of 1000 quills at $\frac{1}{4}$ cent per quill? *Ans.* $2,50*cts.*

5. What is the cost of 900 lead pencils at 6 cents apiece? *Ans.* $54,00.

6. What is the cost of 20*lb.* of soap at $6\frac{3}{4}$*cts.* per pound? *Ans.* $1,35.

7. What is the cost of 140 yards of tape at $2\frac{1}{4}$*cts.* per yard? *Ans.* $3,15.

8. What is the cost of 438 bushels of apples at $31\frac{1}{4}$*cts.* per bushel? *Ans.* $$136,87\frac{1}{2}$.

9. What is the cost of $51\frac{1}{2}$ tons of hay at $12 per ton? *Ans.* $618.

10. What is the cost of 231 yards of linen at 75*cts.* per yard? *Ans.* $173,25.

11. What is the cost of 144*lb.* of rice at $3\frac{1}{2}$*d* per pound? *Ans.* £2 2*s.*

12. What is the cost of $14\frac{1}{4}$ yards of cloth, at $$4\frac{3}{4}$ per yard? *Ans.* $$67,68\frac{3}{4}$.

13. What will 131*lb.* of cheese come to at 1*s* 2*d* per pound? *Ans.* £7 12*s* 10*d.*

14. What will 144 dozen of eggs come to at 1*s* 3*d* per dozen? *Ans.* £9.

15. What will 6*gal.* 1*qt.* 1*pt.* 2*gi.* of wine come to at 5*s* 4*d* per quart. *Ans.* £6 17*s* 4*d.*

16. What will 51 acres of land be worth at £3 2*s* 2*d* per acre? *Ans.* £158 10*s* 6*d.*

17. What will 15*cwt.* 2*qr.* 17*lb.* of sugar come to at 1*s* per pound? *Ans.* £87 13*s.*

18. What will 4E. E. 3*qr.* 2*na.* of broadcloth cost at £2 3*s* 8*d* per yard? *Ans.* £12 16*s* $6\frac{1}{2}$*d.*

19. What will 1*hhd.* 2*gal.* 3*qt.* 1*pt.* 1*gi.* of molasses come to at 12½*cts.* per quart? *Ans.* \$32,95$\frac{5}{16}$.

20. What will be the cost of 27*bu.* 3*pk.* 6*qt.* 1*pt.* of wheat, at 10*s* 2*d* 3*far* per bushel?

*Ans.* £14 5*s* 11*d* 0$\frac{63}{64}$*far.*

---

## SIMPLE INTEREST.

§ **152**. Interest is an allowance made for the use of money that is borrowed.

For example, if I borrow \$100 of Mr. Wilson, for one year, and agree to pay him \$6 for the use of it, the \$6 is called the interest of \$100 for one year, and at the end of the time Mr. Wilson should receive back his \$100 together with the \$6 interest, making the sum of \$106.

The money on which interest is paid, is called the *Principal.*

The money paid for the use of the principal is called the *Interest.*

The principal and interest taken together are called the *Amount.*

In the above example,

\$100 is the principal,
\$ 6 is the interest, and
\$106 is the amount.

The interest of \$100 for one year, determines the rate of interest, or rate per cent. The term per cent, means by the hundred. In the example above the rate of interest is 6 per cent, or \$6 for the hundred. Had \$8 been paid for the use of the \$100, the rate of interest would have been 8 per cent; or had \$3 only been paid, the rate of interest would have been 3 per cent.

The legal interest is the interest established by law.

In the New England States, and indeed in most of the other states, the legal interest is 6 per cent per annum; that is, 6 per cent by the year.

In New York, however, it is 7 and in Louisiana 8 per cent.

Q. What is Interest? What is the money called on which interest is paid? What is the money called which is paid for the use of the principal? What is the amount? What determines the rate of interest? What is the meaning of per cent? What is legal interest? What is meant by per annum? How much is the interest per annum in most of the states? What is it in New York?

CASE I.

§ **153**. To find the interest on any given principal for one or more years.

EXAMPLES.

1. What is the interest of $650 for one year at 6 per cent?

It is plain, that $100 is to $6, its interest for one year, as $650 to its interest for the same time. The fourth term is found by multiplying the second and third terms together and dividing by the first § **140**.

OPERATION.

```
 $     $       $
100 : 6 : : 650 : Ans.
                6
          100)3900
       Ans. $39,00.
```

2. What is the interest on $950 for four years at 7 per cent per annum?

We first find the interest for one year, and then multiply it by the number of years.

OPERATION.

```
 $950
    7
$66,50 interest for 1 year.
     4 number of years.
$266,00         Ans. $266,00.
```

Hence, we have the following

RULE.

I. *Multiply the principal by the rate of interest, and divide the product by* 100: *the quotient will be the interest for one year.*

II. *When the number of years exceeds one, multiply the interest for one year by the number of years: the product will be the interest for that number of years.*

3. What is the interest on $3675 for three years, at 7 per cent per annum? *Ans*. $771,75.

4. What is the interest on $459 for five years at 8 per cent per annum? *Ans*. $183,60.

5. What is the interest of $327,45 for one year at 6 per cent per annum?

We first multiply the given sum $327,45 by the rate of interest, 6 per cent, leaving two places for decimals or cents at the right hand. Then to divide by 100 we remove the decimal point two places to the left, leaving four places for decimals to the right hand. If the principal had contained mills, there would have been five places of decimals in the answer.

OPERATION.

```
       $327,45
             6
  100)1964,70
       19,6470
Ans. 19dolls. 64cts. 7m.
```

Hence, we have the following

RULE.

I. *When the principal contains dollars only, multiply by the rate of interest, and point off two places from the right hand for cents: the places on the left will express the dollars.*

II. *When the principal contains dollars and cents, multiply as before, and strike off four places from the right hand for cents and mills.*

III. *When the principal contains dollars cents and mills, multiply as before and strike off in the product five places from the right hand: the places to the left will be dollars.*

Q. When the principal is in dollars, how do you find the interest for one year? How do you find the interest for two or more years? When the principal contains dollars and cents, how do you find the interest? When the principal contains dollars cents and mills, how do you find the interest?

6. What is the interest on $375, 27*cts.* 3*m.* for two years, at 7 per cent per annum?

We first find the interest for one year, and then multiply by 2. We omit in the answer the decimal figures which fall on the right of the cents.

OPERATION.

```
 $375,273
        7
 $26,26911  interest for 1 year.
        2
 $52,53822
            Ans. $52,53+.
```

7. What is the interest on $211,26 for one year at $4\frac{1}{2}$ per cent per annum?

We first find the interest at 4 per cent, and then the interest for $\frac{1}{2}$ per cent: the sum is the interest at $4\frac{1}{2}$ per cent.

OPERATION.

$$\begin{array}{rl} \$211,26 & \\ 4 & \text{per cent.} \\ \hline 84504 & \\ 10563 & \tfrac{1}{2} \text{ per cent.} \\ \hline \$9,5067 & \\ \hline \end{array}$$

*Ans.* $9,50+.

8. What is the interest on $1576,91 for 3 years at 7 per cent? *Ans.* $331,15+.

9. What is the interest on $957,08 for 6 years at $3\frac{1}{2}$ per cent? *Ans.* $200,98+.

10. What is the interest on $375,45 for 7 years at 7 per cent per annum? *Ans.* $183,970+.

11. What is the interest on $4049,87 for 2 years at 5 per cent per annum? *Ans.* $404,98+.

CASE II.

§ **154**. To find the interest at 6 per cent per annum for any number of months.

At six per cent per annum, each month produces $\frac{1}{2}$ per cent on the principal; and every *two months* produces one per cent on the principal. Therefore, to find the interest for months we have the following

RULE.

*Multiply the principal by half the number of months, and remove the separating point in the product two places farther to the left hand.*

EXAMPLES.

1. What is the interest of $327 for 8 months at 6 per cent per annum?

We here multiply by half the number of months, and the product divided by 100 is the interest.

OPERATION.

$$\begin{array}{rl} \$327 & \\ 4 & \text{half the number of months.} \\ \hline \$13,08 & \\ \hline \end{array}$$

*Ans.* $13,08.

2. What is the interest of $327,47 for 9 months at 6 per cent per annum?

We first multiply by 4 and then add half the multiplicand.

OPERATION.

$327,47
$4\frac{1}{2}$ half the months.
130988
$16373\frac{1}{2}$
$14,7361\frac{1}{2}$ *Ans.* $14,73+.

3. What is the interest on $8975 for ten months at 6 per cent per annum? *Ans.* $448,75.

4. What is the interest on $8753,65 for fourteen months at 6 per cent per annum? *Ans.* $612,7555.

5. What is the interest on $37596,42 for sixteen months at 6 per cent per annum? *Ans.* $3007,7136.

6. What is the interest on $3976,85 for nine months at 6 per cent per annum? *Ans.* $178,9582.

Q. How do you find the interest at 6 per cent for any number of months? What per cent will two months give? Four months? Five months? Six months? Seven months? Eight months? Nine months? Eleven months? Twelve months?

CASE III.

§ **155**. To find the interest at 6 per cent per annum, for any number of days.

In computing interest the month is reckoned at 30 days. Hence, 60 days, which make two months, will give an interest of one per cent on the principal. This one per cent is found by simply removing the separating point two places to the left. If this one per cent be then divided by 60 the quotient will be the interest for 1 day, and this quotient multiplied by the number of days will give the interest required. To divide by 60 we remove the separating point one place farther to the left, and then divide by 6.

Hence, we have the following

RULE.

I. *Remove the separating point in the principal three places to the left, then divide by* 6 *and the quotient will be the interest for* 1 *day.*

II. *Multiply the interest by the number of days, and the product will be the answer sought.*

EXAMPLES.

1. What is the interest on $327,30 for 25 days?

We first remove the decimal point three places to the left; then divide by 6, and afterwards multiply by 25.

OPERATION.

```
6),32730
  ,05455 interest for one day.
      25 number of days.
   27275
  10910
$1,36375        Ans. $1,36+.
```

2. What is the interest on $27,25 for 19 days?

When there are not three figures in the principal, on the left of the separating point, ciphers must be prefixed to supply the deficiency.

OPERATION.

```
6),02725
  ,00454+
      19
   04086
  00454
$0,08626
     Ans. ,08cts.+.
```

3. What is the interest on $575,72 for 29 days? *Ans.* $2,78+.
4. What is the interest on $195,19 for 7 days? *Ans.* $0,22+.
5. What is the interest on $897,04 for 27 days? *Ans.* $4,0366+.
6. What is the interest on $378,53 for 18 days? *Ans.* $1,135+.
7. What is the interest on $885,62 for 25 days? *Ans.* 3,69+.
8. What is the interest on $3756,25 for 17 days? *Ans.* $10,642+.

Q. In computing interest, how many days are reckoned to the month? How much will 60 days produce? How do you find the 1 per cent? If you then divide by 60, what is the quotient? Give the rule for computing the interest for days? When there are not three figures in the principal on the left of the decimal point, what do you do?

### CASE IV.

§ **156.** To find the interest at 6 per cent per annum for years, months, and days.

### RULE.

*Find the interest for the years by Case* I, *for the months by Case* II, *and for the days by Case* III, *then add the several results together and their sum will be the answer sought.*

### EXAMPLES.

1. What is the interest on $1597,27 at 6 per cent for 3 years 9 months and 11 days?

| | | |
|---|---|---|
| $1597,27 | $1597,27 | 6)1,59727 |
| 6 | 4½ | ,26621+ |
| 95,8362 | 638908 | 11 |
| 3 *yr.* | 79863½ | 2,92831 |
| $287,5086 | $71,8771½ | |

| | |
|---|---|
| Interest for 3 years | $287,508+ |
| - - - - - 9 months | 71,877+ |
| - - - - - 11 days | 2,928+ |
| Total interest | $362,313 |

2. What is the interest of $11759,10 at 6 per cent for 9 years 11 months and 16 days? *Ans.* $7028,02+.

3. What is the interest on $9787 for 12 years and 1 day? *Ans.* $7048,27+.

4. What is the interest of $87601,29 for 1 year 1 month and 1 day? *Ans.* $5708,68+.

5. What is the interest of $806,90 for 1 year and 10 months at 6 per cent per annum? *Ans.* $88,75+.

6. What is the interest of $450,75 for 4 years and 7 months at 6 per cent per annum? *Ans.* $123,95+.

7. What is the interest of $443,50 for 7 years 2 months and 12 days at 6 per cent per annum? *Ans.* $191,59+.

8. What will be the total amount of $649,22 after 10 years and 10 months at an interest of 6 per cent? *Ans.* $1071,21+

Q. How do you find the interest for years months and days?

### CASE V.

**§ 157.** When there are months and days, and the rate of interest is greater or less than 6 per cent.

### RULE.

*Find the interest at 6 per cent. Then add to it or subtract from it such a part of the interest so found as the given rate exceeds or falls short of six per cent per annum.*

### EXAMPLES.

1. What is the interest of $119,50 at 7 per cent per annum for 3 years and 4 months?

| | | | |
|---|---|---|---|
| $119,50 | | $119,50 | |
| 2 | half the months | 6 | |
| $2,3900 | int. for 4 months | 7,1700 | |
| | | 3 | |
| | | 21,5100 | int. for 3 years. |
| | | 2,3900 | int. for 4 mo. |
| Total interest at 6 per cent | | $23,9000 | |
| Add one-sixth - - - - - - | | 3,9833+ | |
| Total interest at 7 per cent | | $27,8833+ | |

2. What is the interest on $487,25 for 4 years and 9 months at 4 per cent?

| | | | |
|---|---|---|---|
| $487,25 | | $487,25 | |
| 4½ | half the months | 6 | |
| 194900 | | $29,2350 | |
| 24362+ | | 4 | years |
| $21,9262 | int. for 9 mo. | 116,9400 | int. for 4 years. |
| | | 21,9262 | int. for 9 mo. |
| Total interest at 6 per cent | | $138,8662 | |
| Subtract one-third - - - | | 46,2887+ | |
| Total interest at 4 per cent | | $92,5775+ | |

3. What is the interest of $987,99, at 5 per cent, for 5 years 2 months and 9 days? *Ans.* $256,46+.

4. What is the interest on $437,21, at 3 per cent, for 9 years and 9 months? *Ans.* $127,88+.

5. What is the interest of $15000 for 8 months at 7 per cent per annum? *Ans.* $700.

6. What is the interest of $400 for 21 days at 5 per cent per annum? *Ans.* $1,16+.

7. What is the interest of $876,48, at 7 per cent, for 4 years 9 months and 14 days? *Ans.* $293,815+.

8. What will be the total amount of $1119,48, after 2 years and a half, at an interest of 7 per cent per annum? *Ans.* $1315,389+.

9. What is the interest on $532,41 for 3 years and 3 months at $4\frac{1}{2}$ per cent per annum? *Ans.* $77,86+.

10. What is the interest on $8375,27, at 5 per cent per annum, for 5 years 5 months and 5 days? *Ans.* $2274,118+.

11. What is the interest of $8759,27, at 6 per cent per annum, for 1 year 6 months and 9 days? *Ans.* $801,473+.

12. What is the interest, at $6\frac{1}{2}$ per cent per annum, on $7569,11, for 3 years 4 months and 18 days? *Ans.* $1664,573+.

Q. When the rate of interest is greater or less than 6 per cent, how do you find the interest for months and days?

§ **158.** Note. In computing interest, it is often very convenient to find the interest for the months by considering them as aliquot parts of a year, and the interest for the days by considering them as aliquot parts of a month.

EXAMPLES.

1. What is the interest of $806,90 for one year 10 months and 10 days at 6 per cent?

| | | |
|---|---|---|
| $806,90 | | |
| 6 | | |
| 6)$48,4140 | int. for 1 year. | $8,069 |
| 2)8,069 | int. for 2 months. | 5 |
| 3)4,034+ | int. for 1 month. | $40,345 int. for 10 mo. |
| 1,344+ | int. for 10 days. | |

| | |
|---|---|
| Interest for 1 year, - - | $48,4140 |
| - - - - - 10 months - | 40,345 |
| - - - - - 10 days - - | 1,344+ |
| Total interest | $90,103+ |

2. What is the interest of \$200 for 10 years 3 months and 6 days at 7 per cent?

```
     $200
        7
 4)14,00   int. for 1 year.           $14,00
  3)3,50   int. for 3 months.             10
  5)1,16+  int. for 1 month.         $140,00 for 10 years.
    ,23+   int. for 6 days.

          $140,00   interest for 10 years.
             3,50   interest for 3 months.
              ,23+  interest for 6 days.
   Ans.   $143,73+.
```

3. What is the interest of \$132,26 for 1 year 4 months and 10 days, at 6 per cent per annum? *Ans.* \$10,80+.

4. What is the interest of \$25,50 for 1 year 9 months and 12 days, at 6 per cent? *Ans.* \$2,72+.

5. What is the interest of \$347,25 for 1 year 1 month and 6 days, at 4 per cent per annum? Also, at 5 per cent? At 5½ per cent? At 6 per cent? At 7 per cent? At 7½ per cent? At 8 per cent? At 8½ per cent? And at 9 per cent?

6. What is the interest, at 6 per cent per annum, on \$48,32, for 1 year 1 month and 15 days? *Ans.* \$3,26+.

7. What is the interest, at 8 per cent per annum, on \$675,87, for 3 years 6 months and 6 days?

*Ans.* \$190,14+.

8. What is the interest, at 7 per cent, on \$587,25, for 5 years 5 months and 5 days? *Ans.* \$223,23+.

9. What is the interest on \$67589,20 for 3 years 9 months and 12 days, at 5 per cent per annum?

*Ans.* \$12785,62+.

### CASE VI.

§ **159**. When the sum on which the interest is to be cast is in pounds, shillings, and pence.

### RULE.

I. *Reduce the shillings and pence to the decimal of a pound* (see § **135**).

II. *Then find the interest as though the sum were dollars and cents; after which reduce the decimal part of the answer to shillings and pence* (see § **137**).

EXAMPLES.

1. What is the interest, at 6 per cent, of £27 15*s* 9*d* for 2 years?

We first find the interest for one year. We then multiply by 2, which gives the interest for two years. We then reduce to pounds shillings and pence.

OPERATION.

| |
|---:|
| £27 15*s* 9*d*=£27,7875 |
| 6 |
| 1,667250 |
| 2 |
| £3,334500 |
| 20 |
| 6,690000 |
| 12 |
| 8,280000 |
| 4 |
| 1,120000 |

*Ans.* £3 6*s* 8¼*d*+.

2. What is the interest on £67 19*s* 6*d*, at 6 per cent, for 3 years 8 months 16 days? *Ans.* £15 2*s* 8½*d*+.

3. What is the interest on £127 15*s* 4*d*, at 6 per cent, for 3 years and 3 months? *Ans.* £24 18 3¼*d*+.

4. What is the interest of £107 16*s* 10*d*, at 6 per cent, for 3 years 6 months and 6 days? *Ans.* £22 15*s* 1*d*+.

5. What will £279 13*s* 8*d* amount to in 3 years and a half, at 5¼ per cent per annum? *Ans.* £331 1*s* 6*d*+.

6. What is the interest of £514 10*s* 2*d* for 3 years and a half, at 4 per cent? *Ans.* £72 0*s* 7¼*d*+.

7. What is the interest of £523 11*s* 6*d* for 3 years and a half at 6 per cent? *Ans.* £109 19 0*d*+.

8. What is the interest on £255 10*s* 8*d* at six per cent per annum, for 6*yr.* 6*mo.*? *Ans.* £99 13*s* 1¾*d*.

9. What is the interest on £53 18*s* 5*d* at 6 per cent for 7*yr.* 12*da.*? *Ans.* £22 15*s* 1*d*+.

APPLICATIONS.

Calculate the interest on the following notes.

$127,50 New York, January, 1st 1838.

1. For value received I promise to pay on the 10th day of June next, to Wm. Johnson or order, the sum of one hundred and twenty-seven dollars and fifty cents with interest from date, at 7 per cent. *John Liberal.*

*Ans.* $131,46+.

$306 New York, January 1st, 1833.

2. For value received I promise to pay on the 4th of July, 1835, to Wm. Johnson or order, three hundred and six dollars with interest at 6 per cent from the 1st of March, 1833. *John Liberal.*

*Ans.* $349,04+.

$1040 Hartford, July, 3rd 1837.

3. Six months after date, I promise to pay to C. Jones or order, one thousand and forty dollars with interest from the 1st of January last, at 7 per cent. *Joseph Springs.*

*Ans.* $1113,40+.

§ **160**. We shall now give the rule established in New York, (See Johnson's Chancery Reports, Vol. I. page 17,) for computing the interest on a bond or note, when partial payments have been made. The same rule is also adopted in Massachusetts, and in most of the other states.

RULE.

I. *Compute the interest on the principal to the time of the first payment, and if the payment exceed this interest, add the interest to the principal and from the sum subtract the payment: the remainder forms a new principal.*

II. *But if the payment is less than the interest, take no notice of it until other payments are made, which in all, shall exceed the interest computed to the time of the last payment: then add the interest, so computed, to the principal, and from the sum subtract the sum of the payments: the remainder will form a new principal on which interest is to be computed as before.*

### EXAMPLES.

$349,99 8. May 1st, 1826.

1. For value received I promise to pay James Wilson or order, three hundred and forty-nine dollars ninety-nine cents and eight mills with interest, at 6 per cent.

*James Paywell.*

On this note were endorsed the following payments:

| | | | |
|---|---|---|---|
| Dec. | 25th, 1826 | Received | $49,998 |
| July | 10th, 1827 | " | $ 4,998 |
| Sept. | 1st, 1828 | " | $15,000 |
| June | 14th, 1829 | " | $99,999 |

What was due April 15th, 1830?

| | | |
|---|---|---|
| Principal on int. from May 1st, 1826, . . . | | $349,998 |
| Interest to Dec. 25th, 1826, time of first payment, 7 months 24 days. . . . | | 13,649+ |
| Amount . . . | | $363,647 |
| Payment Dec. 25th, exceeding interest then due . . . . . . . . . . . | | $ 49,998 |
| Remainder for a new principal . . . . | | $313,649 |
| Interest of $313,649 from Dec. 25th, 1826, to June 14th, 1829, 2 years 5 months 19 days . . . . . . . . . . . . . | | $ 46,472+ |
| Amount . . . | | $360,121 |
| Payment, July 10th, 1827, less than interest then due . . | $ 4,998 | |
| Payment, Sept. 1st, 1828 . . . | 15,008 | |
| Their sum . . . . . . . less than interest then due | $20,006 | |
| Payment, June 14th, 1829 . . | 99,999 | |
| Their sum exceeds the interest then due | | $120,005 |
| Remainder for a new principal, June 14th, 1829 . . . . . . . . . . . . . | | $240,116 |
| Interest of $240,116 from June 14th, 1829, to April 15th, 1830, 10 months 1 day . | | 12,045 |
| Total due, April 15th, 1830 . . | | $252,161+. |

$3469,32.

2. For value received, I promise to pay WILLIAM JENKS, or order, three thousand four hundred and sixty-nine dollars and thirty-two cents, with interest from date, at 6 per cent. Feb. 6th, 1825. BILL SPENDTHRIFT.

On this note were endorsed the following payments:—

May 16th, 1828, received $545,76.
May 16th, 1830, received $1276.
Feb. 1st, 1831, received $2074,72.

What remained due August 11th, 1832?

*Ans.* $860,55+.

3. A's note of $635,84 was dated Sept. 5th, 1817, on which were endorsed the following payments, viz:— Nov. 13th, 1819, $416,08; May 10th, 1820, $152: what was due March 1st, 1821, the interest being 6 per cent?

*Ans.* $168,01+.

---

## COMPOUND INTEREST.

§ **161**. Compound Interest is when the interest on a sum of money becoming due, and not being paid, is added to the principal, and the interest then calculated on this amount, as on a new principal. For example, suppose I were to borrow of Mr. Wilson $200 for one year, at 6 per cent, and at the end of the year pay him neither the interest nor principal. Now if Mr. Wilson should add the interest, $12, to the principal, $200, making $212, and charge me with interest on this sum till I paid him, this would be Compound Interest, because it is interest upon interest.

### RULE.

*Calculate the interest to the time at which it becomes due: then add it to the principal and calculate the interest on the amount as on a new principal: add the interest again to the principal and calculate the interest as before: do the same for all the times at which payments of interest become due: from the last result subtract the principal, and the remainder will be the compound interest.*

EXAMPLES.

1. What will be the compound interest, at 7 per cent, of $3750 for 4 years, the interest being added yearly?

| | | |
|---|---|---|
| | $3750,00 | principal for 1st year. |
| $3750 × 7 ÷ 100 = | 262,50 | interest for 1st year. |
| | 4012,50 | principal for 2nd „ |
| $4012,50 × 7 ÷ 100 = | 280,87+ | interest for 2nd „ |
| | 4293,37+ | principal - 3rd „ |
| $4293,37 × 7 ÷ 100 = | 300,53+ | interest - 3rd „ |
| | 4593,90+ | principal - 4th „ |
| $4593,90 × 7 ÷ 100 = | 321,57+ | interest - 4th „ |
| | 4915,47+ | amount at 4 years. |
| 1st principal | 3750,00 | |
| amount of interest. | $1165,47+ | |

2. If the interest be computed annually, what will be the interest on $100 for three years, at 6 per cent?
*Ans.* $19,101+.

3. What will be the compound interest on $295,37, at 6 per cent, for 2 years, the interest being added annually?
*Ans.* $36,50+.

4. What will be the compound interest on $500 for one year, at 8 per cent, the interest being computed quarterly?
*Ans.* $41,21+.

**Q. What is Compound Interest? Give the Rule for computing Compound Interest?**

---

## COMMISSION AND BROKERAGE.

§ **162**. Commission is an allowance made to a factor or commission merchant for buying and selling. Brokerage is an allowance made to dealers in money or stocks. The allowance made is generally a certain per cent, or rate per hundred, on the moneys paid out or received, and the amount may be determined by the rules of simple interest.

### EXAMPLES.

1. What is the commission on $4396 at 6 per cent?

We here find the commission, as in simple interest, by multiplying by the rate per cent and dividing by 100.

OPERATION.

```
 $4396
     6
 -----
$263,76
```

*Ans.* $263,76.

2. A factor sells 60 bales of cotton at $425 per bale, and is to receive 2½ per cent commission: how much must he pay over to his principal? *Ans.* $24862,50.

3. A sent to B, a broker, $3825 to be invested in stock: B is to receive 2 per cent on the amount paid for the stock: what was the value of the stock purchased?

As B is to receive 2 per cent, it follows that $102 of A's money will purchase $100 of stock: hence 100+ the commission, is to 100, as the given sum to the stock which it will purchase.

OPERATION.

```
100
  2
---
102 : 100 :: 3225
               100
      ------------
      102)382500(3750
          306
          ---
           765
           714
           ---
            510
            510
```

*Ans.* $3750.

PROOF.

| | |
|---|---|
| | $3750 |
| Commission on $3750, at 2 per cent= | 75 |
| Total Sum . . . | $3825 |

4. A factor receives $708,75, and is directed to purchase iron at $45 per ton: he is to receive 5 per cent on the money paid: how much iron can he purchase?

*Ans.* 15*tons.*

5. Messrs. P, W and K buy 200 shares of United States stock for Mr. A. They pay $197 per share, and

are to receive one-fourth per cent on the money they advance: how much must A pay them for the stock?
*Ans.* $39498,50.

6. Messrs. P, W and K receive $28750 to be invested in stock. They charge 2½ per cent commission on the amount paid: what is the value of the stock purchased?
*Ans.* $28048,78+.

7. The par value or first cost of 167 shares of bank stock was $200 per share: what is the present value, if the stock is at a premium of 25 per cent, that is, 25 per cent above par. *Ans.* $41750.

8. What would be the value of the stock named in the last example, if it were at a discount of 10 per cent?
*Ans.* $30060.

9. One hundred shares of United States Bank stock is worth 18½ per cent premium: the par value being $200 per share, what is the value of the stock? *Ans.* $23700.

10. A bank fails, and has in circulation bills to the amount of $267581. It can pay 9½ per cent: how much money is there on hand? *Ans.* $25420,19½.

11. Sixty-nine shares of bank stock, of which the par value is $125, is at a discount of 8 per cent: what is its value? *Ans.* $7935.

Q. What is commission? What is brokerage? How is the allowance generally made? How is the commission or brokerage found? How do you find the amount of stock to be purchased when the broker receives a certain per cent on the amount purchased, as in Example 3?

---

## INSURANCE.

§ **163**. Insurance is an agreement by which an individual or a company agrees to exempt the owners of certain property from loss or hazard.

The written agreement is called the *policy*.

The premium is the amount paid by him who owns the property, to those who insure it, as a compensation for their risk. It is generally so much *per cent* on the value of the property insured.

EXAMPLES.

1. What would be the premium for the insurance of a house valued at $5500 against loss by fire for 1 year, at $\frac{1}{2}$ per cent.

By dividing by 100, we have the insurance at 1 per cent. . - - - - - - - } 55,00.

The half, is the insurance at half per cent. $27,50.

2. What would be the premium for insuring a ship and cargo, valued at $37500 from New York to Liverpool, at $3\frac{1}{2}$ per cent? *Ans.* $1312,50.

3. What would be the insurance on a ship valued at $47520 at $\frac{1}{2}$ per cent: also at $\frac{1}{3}$ per cent?

*Ans.* $237,60.—$158,40.

4. What would be the insurance on a house valued at $14000 at $1\frac{1}{2}$ per cent? Also, at $\frac{3}{4}$ per cent? At $\frac{1}{2}$ per cent? At $\frac{1}{3}$ per cent? At $\frac{1}{4}$ per cent?

*Ans.* $210.—$105.—$70.—$46,66+.—$35.

5. What is the insurance on a store and goods, valued at $27000, at $2\frac{1}{4}$ per cent? At 2 per cent? At $1\frac{1}{2}$ per cent? At $\frac{3}{4}$ per cent? At $\frac{1}{2}$ per cent? At $\frac{1}{4}$ per cent? At $\frac{1}{5}$ per cent? At $\frac{1}{6}$ per cent?

**Q. What is insurance? What is the policy? What is the premium? How is it generally reckoned?**

---

## DISCOUNT.

§ **164**. If I give my note to Mr. Wilson for $106, payable in one year, the present value of the note will be less than $106 by the interest on its *present value* for one year: that is, its present value will be $100.

The amount named in a note is called the *face of the note.* Thus $106 is the face of the note to Mr. Wilson.

The present value of a note is that sum which being put at interest until the note becomes due would increase to an amount equal to the face of the note. Thus $100 is the present value of the note to Mr. Wilson.

*The discount is the difference between the face of a note and its present value.* Thus, $6 is the discount on the note to Mr. Wilson.

### RULE.

*As* 100+ *interest of* $100 *for the given time, is to* 100, *so is the face of the note to its present value.*

### EXAMPLES.

1. What is the present value of a note for $1828,75 due in one year, at $4\frac{1}{2}$ per cent per annum?

```
  100
    4,50  interest of $100 for the time.
  104,50 : 100 :: 1828,75 : Ans.
                             100
               104,50)182875,00($1750.
```

*Ans.* $1750.

2. What is the present value of a note for $1290,81 discounted for four months, at 6 per cent per annum?
*Ans.* $1265,50.

5. What is the present value of $800, due 4 years hence: the interest being computed at 5 per cent per annum? *Ans.* $666,66 6+.

NOTE. When payments are to be made at different times, *find the present value of the several sums separately and their sum will be the present value of the note.*

4. What is the present value of a note for $3500 on which $300 are to be paid in 6 months; $900 in one year; $1300 in eighteen months; and the residue at the expiration of two years: the rate of interest being 6 per cent per annum? *Ans.* $3225,83+.

5. What is the discount of £1500 one-half payable in 6 months and the other half at the expiration of a year, at 7 per cent per annum? *Ans.* £74 8*s* $6\frac{3}{4}$*d*+.

6. What is the present value of $2880, one-half payable in 3 months, one-third in 6 months, and the rest in 9 months at 6 per cent per annum? *Ans.* $2810,08+.

Q. What is the face of a note? What is the present value of a note? What is the discount of a note? How do you find the present value of a note? When payments are to be made at different times, how do you find the present value?

## LOSS AND GAIN.

§ **165**. Loss and Gain is a rule by which merchants discover the amount lost or gained in the purchase and sale of goods. It also instructs them how much to increase or diminish the price of their goods so as to make or lose so much per cent.

### EXAMPLES.

1. Bought a piece of cloth containing 75*yd.* at $5,25 per yard, and sold it at $5,75 per yard: how much was gained in the trade?

OPERATION.

We first find the profit on a single yard, and then the profit on the 75 yards.

$5,75 price of 1 yard.
$5,25 cost of 1 yard.
50*cts.* profit on 1 yard.

*yd.* *yd.* *cts.*
1 : 75 :: 50 : *Ans.*
75
$37,50

*Ans.* $37,50.

2. Bought a piece of calico containing 50*yd.* at 2*s* 6*d* per yard: what must it be sold for per yard to gain £1 0*s* 10*d*?

50*yd.* at 2*s* 6*d*=£6 5*s*
Profit =£1 0*s* 10*d*
It must sell for £7 5*s* 10*d*
50)£7 5*s* 10*d*(2*s* 11*d*

*Ans.* 2*s* 11*d*.

3. Bought a hogshead of brandy at $1,25 per gallon, and sold it for $78: was there a loss or gain?

*Ans.* loss of $0,75.

4. A merchant purchased 3275 bushels of wheat for which he paid $3517,10, but finding it damaged is willing to lose 10 per cent: what must he sell it for per bushel?

*Ans.* $0,96+.

5. A bought a piece of cotton containing 40 yards, at 6 cents per yard; he sold it for $7\frac{1}{2}$ cents per yard: how much did he gain? *Ans.* $0,60.

6. Bought a piece of cloth containing 75 yards for $375: what must it be sold for per yard, in order to gain $100? *Ans.* $6,33⅓ *per yard.*

7. Bought a quantity of wine at $1,25 per gallon, but it proves to be bad and am obliged to sell it at 20 per cent less than I gave: how much must I sell it for per gallon? *Ans.* $1 *per gall.*

8. A farmer sells 125 bushels of corn for 75*cts.* per bushel; the purchaser sells it at an advance of 20 per cent: how much did he receive for the corn? *Ans.* $112,50.

9. A merchant buys one tun of wine for which he pays $725, and wishes to sell it by the hogshead at an advance of 15 per cent: what must he charge per hogshead? *Ans.* $208,43+.

10. A merchant buys 158 yards of calico for which he pays 20 cents per yard; one-half is so damaged that he is obliged to sell it at a loss of 6 per cent; the remainder he sells at an advance of 19 per cent: how much did he gain? *Ans.* $2,05+.

---

## EQUATION OF PAYMENTS.

§ **166**. I owe Mr. Wilson $2 to be paid in 6 months; $3 to be paid in 8 months; and $1 to be paid in 12 months. I wish to pay his entire dues at a single payment, to be made at such a time, that neither he nor I shall lose interest: at what time must the payment be made?

The method of finding the mean time of payment of several sums due at different times, is called *Equation of Payments.*

Taking the example above.

| | | | | | |
|---|---|---|---|---|---|
| Int of | $2 | for 6*mo.*=int. of $1 | for 12*mo.* | $1\times 6=12$ |
| " of | $3 | for 8*mo.*=int. of $1 | for 24*mo.* | $3\times 8=24$ |
| " of | $1 | for 12*mo.*=int. of $1 | for 12*mo.* | $1\times 12=12$ |
| | $6 | | 48 | 48 |

The interest on all the sums, to the times of payment, is equal to the interest of $1 for 48 months. But 48 is equal to the sum of all the products which arise from multiplying each sum by the time at which it becomes due: hence, the sum of the products is equal to the time which would be necessary for $1 to produce the same interest as would be produced by all the sums.

Now, if $1 will produce a certain interest in 48 months, in what time will $6 (or the sum of the payments) produce the same interest. The time is obviously found by dividing 48, (the sum of the products), by $6, (the sum of the payments).

Hence, we have the following

### RULE.

*Multiply each payment by the time before it becomes due, and divide the sum of the products by the sum of the payments: the quotient will be the mean time.*

2. B owes A $600: $200 is to be paid in two months, $200 in four months, and $200 in six months: what is the mean time for the payment of the whole?

We here multiply each sum by the time at which it becomes due, and divide the sum of the products by the sum of the payments.

OPERATION.

$$\begin{array}{r} 200\times2=\ \ 400 \\ 200\times4=\ \ 800 \\ 200\times6=1200 \\ \hline 600\ \ \ \ )24|00 \\ \hline 4 \end{array}$$

*Ans.* 4 *months.*

3. A merchant owes $600, of which $100 is to be paid in 4 months, $200 in 10 months, and the remainder in 16 months: if he pays the whole at once, at what time must he make the payment? *Ans.* 12 *months.*

4. A merchant owes $600 to be paid in 12 months, $800 to be paid in 6 months, and $900 to be paid in 9 months: what is the equated time of payment.

*Ans.* 8*mo.* $22\frac{4}{23}$*da.*

5. A owes B $600; one-third is to be paid in 6 months, one-fourth in 8 months, and the remainder in 12 months: what is the mean time of payment? *Ans.* 9 *months.*

6. A merchant has due him $300 to be paid in 60 days, $500 to be paid in 120 days, and $750 to be paid in 180 days: what is the equated time for the payment of the whole? *Ans.* $137\frac{13}{31}$ *days.*

7. A merchant has due him $1500; one-sixth is to be paid in 2 months; one-third in 3 months; and the rest in 6 months: what is the equated time for the payment of the whole? *Ans.* $4\frac{1}{3}$ *months.*

NOTE. If one of the payments is due on the day from which the equated time is reckoned, its corresponding product will be nothing, but the payment must still be added in finding the sum of the payments.

8. I owe $1000 to be paid on the 1st of January, $1500 on the 1st of February, $3000 on 1st of March, and $4000 on the 15th of April: reckoning from the 1st of January, and calling February 28 days, on what day must the money be paid?

*Ans. Payment in* $67\frac{14}{19}$ *days, or on the* 8*th March.*

Q. What is Equation of Payments? What is the sum of the products which arise from multiplying each payment by the time to which it becomes due equal to? How do you find the time of mean payment? When you reckon the time from the date at which the first first payment becomes due, do you include the first payment?

---

## FELLOWSHIP.

§ **167**. Fellowship is the joining together of several persons in trade with an agreement to share the losses and profits according to the amount which each one puts into the partnership. The money employed is called the *Capital Stock.*

The gain or loss to be shared is called the *Dividend.*

It is plain that the whole stock which suffers the gain or loss must be to gain or loss, as the stock of any individual to his part of the gain or loss.

Hence, we have the following

### RULE.

*As the whole stock is to the whole gain or loss, so is each man's share to his share of the gain or loss.*

Q. What is Fellowship? What is the gain or loss called? What is the rule for finding each one's share?

### EXAMPLES.

1. A and B buy certain merchandise amounting to £160, of which A pays £90, and B £70: they gain by the purchase £32: what is each one's share of the profits?

A . . £90
B . . £70

£160 : 32 :: { 90 : £18 A's share.
70 : £14 B's share. }

2. A and B have a joint stock of $2100, of which A owns $1800 and B $300: they gain in a year $1000: what is each one's share of the profits?

*Ans.* A's=$857,14+; B's=$142,85+.

3. A, B, C and D have £20,000 in trade: at the end of a year their profits amount to £16,000: what is each one's share, supposing A to receive £50 and D £30 out of the profits for extra services?

*Ans.* { A's=£4030; B's=£3980;
C's=£3980; D's=£4010. }

4. Five persons, A, B, C, D and E have to share between them an estate of $10,000: A is to have one-fourth; B one-eighth; C one-sixth; D one-eighth; and E what is left: what will be the share of each?

*Ans.* A's=$2500; B's=$1250; C's=$1666,66+;
D's=$1250; E's=$3333,34.

### PROOF.

Add all the separate profits or shares together; their sum should be equal to the gross profit or stock.

## DOUBLE FELLOWSHIP.

§ **168.** When several persons who are joined together in trade employ their capital for different periods of time, the partnership is called *Double Fellowship.*

For example, suppose A puts $100 in trade for 5 years; B $200 for 2 years, and C $300 for 1 year: this would make a case of double fellowship.

Now it is plain that there are two circumstances which should determine each one's share of the profits: 1*st, the amount of capital he puts in; and* 2*ndly, the time which it is continued in the business.*

Hence each one's share should be proportional to the capital he puts in, multiplied by the time it is continued in trade. Therefore we have the following

### RULE.

*Multiply each man's stock by the time he continues it in trade: then say, as the sum of the products is to the whole gain or loss, so is each particular product to each man's share of the gain or loss.*

Q. What is Double Fellowship? What two circumstances determine each one's share of the profits? Give the rule finding each one's share?

### EXAMPLES.

1. A and B enter into partnership: A puts in £840 for 4 months, and B puts in £650 for 6 months: they gain £300: what is each one's share of the profits?

A's stock £840 × 4 = 3360
B's stock £650 × 6 = 3900

£7260 : 300 :: { 3360 : £138 16*s* 10*d*.
                 { 3900 : £161 3*s* 1*d*.

2. A put in trade £50 for 4 months, and B £60 for 5 months: they gained £24: how is it to be divided between them? *Ans.* A's share = £9 12*s*; B's = £14 8*s*.

3. C and D hold a pasture together, for which they pay £54: C pastures 23 horses for 27 days, and D 21 horses for 39 days: how much of the rent ought each one to pay? *Ans.* C, £23 5*s* 9*d*; D, £30 14*s* 3*d*.

# TARE AND TRET.

§ **169**. *Tare* and *Tret* are allowances made in selling goods by weight.

*Draft* is an allowance on the gross weight in favour of the buyer or importer: it is always deducted before the *Tare.*

*Tare* is an allowance made to the buyer for the weight of the hogshead, barrel or bag, &c., containing the commodity sold.

*Gross Weight* is the whole weight of the goods, together with that of the hogshead, barrel, bag, &c., which contains them.

*Suttle* is what remains after a *part of* the allowances have been deducted from the gross weight.

*Net Weight* is what remains after all the deductions are made.

Q. What are Tare and Tret? What is Draft? What is Tare? What is Gross Weight? What is Suttle? What is Net Weight?

EXAMPLES.

1. What is the net weight of 25 hogsheads of sugar, the gross weight being 66*cwt.* 3*qr.* 14*lb.*; tare 11*lb.* per hogshead?

| | *cwt.* | *qr.* | *lb.* | |
|---|---|---|---|---|
| | 66 | 3 | 14 | gross. |
| 25 × 11 = 275*lb.* . . . | 2 | 1 | 23 | tare. |
| *Ans.* | 64 | 1 | 19 | net. |

2. If the tare be 4*lb.* per hundred, what will be the tare on 6*T.* 2*cwt.* 3*qr.* 14*lb.*?

| | |
|---|---|
| Tare for 6*T.* or 120*cwt.* = | 480*lb.* |
| 2*cwt.* = | 8 |
| 3*qr.* = | 3 |
| 14 *lb.* = | 0½ |
| Tare . . . . . | 491½ |

*Ans.* 4*cwt.* 1*qr.* 15½*lb.*

3. What is the net weight of 32 boxes of soap, weighing 31550*lb.*, allowing 4*lb.* per box for draft and 12 per cent for tare?

| | | | |
|---|---|---|---|
| | 31550 | gross. | 31422 |
| 32×4= | 128 | draft. | 12 |
| | 31422 | | 3770,64 |

*Ans.* 3770,64*lb.*=1*T.* 13*cwt.* 2*qr.* 18*lb.* 10*oz.*+

4. What will be the cost of 3 hogsheads of tobacco at $9,47 per cwt. net, the gross weight being of

| | *cwt.* | *qr.* | *lb.* | | *lb.* |
|---|---|---|---|---|---|
| No. 1 . . | 9 | 3 | 25 | . . tare | 146 |
| „ 2 . . | 10 | 2 | 12 | . . „ | 150 |
| „ 3 . . | 11 | 1 | 25 | . . „ | 158 |

*Ans.* $265,16.

5. At £1 5*s* per cwt. net; tare 4*lb.* per cwt.: what will be the cost of 4 hogsheads of sugar weighing gross,

| | *cwt.* | *qr.* | *lb.* | |
|---|---|---|---|---|
| No. 1 . . . | 10 | 3 | 6 | |
| „ 2 . . . | 12 | 5 | 19 | |
| „ 3 . . . | 13 | 1 | 10 | |
| „ 4 . . . | 11 | 2 | 7 | |
| | 49 | 0 | 14 | gross. |
| Tare 4*lb.* per cwt. | 1 | 3 | 0 | 8*oz.* |
| | 47 | 1 | 13 | 8*oz.* net. |

*Ans.* £59 4*s* 3*d*+.

6. At 21 cents per lb., what will be the cost of 5*hhd.* of coffee weighing in gross,

| | *cwt.* | *qr.* | *lb.* | | *lb.* |
|---|---|---|---|---|---|
| No. 1 . . | 6 | 2 | 14 | . . tare | 94 |
| „ 2 . . | 9 | 1 | 20 | . . „ | 100 |
| „ 3 . . | 6 | 2 | 22 | . . „ | 88 |
| „ 4 . . | 7 | 2 | 25 | . . „ | 89 |
| „ 5 . . | 8 | 0 | 13 | . . „ | 100 |

*Ans.* $808,71.

7. At £7 5*s* per cwt. net, how much will 16*hhd.* of sugar come to, each weighing gross 8*cwt.* 3*qr.* 7*lb.*; tare 12*lb.* per cwt.? *Ans.* £912 14*s* 5½*d*+.

8. What is the net weight of 18*hhd.* of tobacco, each weighing gross 8*cwt.* 3*qr.* 14*lb.*; tare 16*lb.* to the cwt.?
*Ans.* 6*T.* 16*cwt.* 3*qr.* 20*lb.*

9. In 4*T.* 3*cwt.* 3*qr.* gross, tare 20*lb.* to the cwt., what is the net weight? *Ans.* 3*T.* 8*cwt.* 3*qr.* 5*lb.*

10. What is the net weight and value of 80 kegs of figs, gross weight 7*T.* 11*cwt.* 3*qr.*, tare 14*lb.* per cwt., at \$2,31 per cwt.?
*Ans.* { 6*T.* 12*cwt.* 3*qr.* 3*lb.* 8*oz.* Value \$306,72 4+.

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## DUODECIMALS.

§ **170.** Duodecimals are denominate fractions in which 1 foot is the unit that is divided.

The unit 1 foot is first supposed to be divided into 12 equal parts, called inches or primes, and marked $'$.

Each of these parts is supposed to be again divided into 12 equal parts, called seconds, and marked $''$.

Each second is divided in like manner into 12 equal parts, called thirds, and marked $'''$.

This division of the foot gives

$1'$ inch or prime . . . . . $=\frac{1}{12}$ of a foot.

$1''$ second is $=\frac{1}{12}$ of $\frac{1}{12}$ . $=\frac{1}{144}$ of a foot.

$1'''$ third is $=\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}=\frac{1}{1728}$ of a foot.

Duodecimals are added and subtracted like other denominate numbers, 12 of a lesser denomination making one of a greater, as in the following

TABLE.

$12'''$ make $1''$ second.
$12''$ . . . $1'$ inch or prime.
$12'$ . . . 1 foot.

EXAMPLES.

1. In $185'$, how many feet? *Ans.* 15*ft.* $5'$.
2. In $250''$, how many feet and inches? *Ans.* 1*ft.* $8'$ $10''$.
3. In $4367'''$, how many feet? *Ans.* 2*ft.* $6'$ $3''$ $11'''$

**Q. In Duodecimals what is the unit that is divided? How is it divided? How are these parts again divided? What are the parts called? How are duodecimals added and subtracted? How many of one denomination make 1 of the next greater?**

EXAMPLES IN ADDITION AND SUBTRACTION.

1. What is the sum of 3*ft.* 6′ 3″ 2‴ and 2*ft.* 1′ 10″ 11‴?
*Ans.* 5*ft.* 8′ 2″ 1‴.

2. What is the sum of 8*ft.* 9′ 7″ and 6*ft.* 7′ 3″ 4‴?
*Ans.* 15*ft.* 4′ 10″ 4‴.

3. What is the difference between 9*ft.* 3′ 5″ 6‴ and 7*ft.* 3′ 6″ 7‴? *Ans.* 1*ft.* 11′ 10″ 11‴.

4. What is the difference between 40*ft.* 6′ 6″ and 29*ft.* 7‴? *Ans.* 11*ft.* 6′ 5″ 5‴.

## MULTIPLICATION OF DUODECIMALS.

§ **171.** It has been shown (§ **64**) that feet multiplied by feet give square feet in the product.

EXAMPLES.

1. Multiply 6*ft.* 6′ 6″ by 2*ft.* 7′.

Set down the multiplier under the multiplicand, so that feet shall fall under feet, inches under inches, &c. It is generally most convenient to begin with the highest denomination of the multiplier, and then multiply first the lower denominations of the multiplicand.

OPERATION.

| *ft.* | | | |
|---|---|---|---|
| 6 | 6′ | 6″ | |
| 2 | 7′ | | |
| 13 | 1′ | | |
| 3 | 9′ | 9″ | 6‴ |
| 16 | 10′ | 9″ | 6‴. |

The 6″ of the multiplicand is $\frac{6}{12}$ of an inch, or $\frac{6}{144}$ of a foot. Therefore when we multiply it by 2 feet, the product is 12″, equal to 1 inch. Multiplying 6′, or $\frac{6}{12}$ of a foot, by 2 feet, the product is 12′, to which add 1 inch from the last product, making 13′. Set down 1′ under the column of inches and carry 1 foot to the product of the 6 by 2, making 13 feet.

Then multiply by 7′. The product of 7′ by 6″=42‴: for, 7′=$\frac{7}{12}$ of a foot, and 6″=$\frac{6}{144}$ of a foot: hence 7′×6″ $=\frac{7}{12}\times\frac{6}{144}=\frac{42}{1728}$=42‴=3″ 6‴. Then $\frac{7}{12}\times\frac{6}{12}=\frac{42}{144}=$

42″, and 3″ to carry make 45″=3′9″: set down 9″. Then $\frac{1}{12}$ by 6=42′, and 3′ to carry make 45′=3*ft.* 9′, which are set down in their proper places.

Hence, we see,

1st, *That feet multiplied by feet give square feet in the product.*

2nd, *That feet multiplied by inches give inches in the product.*

3rd, *That inches multiplied by inches give seconds, or twelfths of inches in the product.*

4th, *That inches multiplied by seconds give thirds in the product.*

2. Multiply 9*ft.* 4*in.* by 8*ft.* 3*in.*

Beginning with the 8 feet, we say 8 times 4 are 32′, which is equal to 2 feet 8′: set down the 8′. Then say 8 times 9 are 72 and 2 to carry are 74 feet: then multiplying by 3′, we say, 3 times 4′ are 12″, equal to 1 inch: set down 0 in the second's place: then 3 times 9 are 27 and 1 to carry make 28′, equal to 2*ft.* 4′. Therefore the entire product is equal to 77*ft.*

OPERATION.

| | | | |
|---|---|---|---|
| 9 | 4′ | | |
| 8 | 3′ | | |
| 74 | 8′ | | |
| 2 | 4′ | 0″ | |
| 77 | 0′ | 0″ | *Ans.* |

3. How many solid feet in a stick of timber which is 25*ft.* 6*in.* long, 2*ft.* 7*in.* broad, and 3*ft.* 3*in.* thick?

It is shown § 65, that the number of solid or cubic feet, is equal to the product of the length, breadth, and thickness.

OPERATION.

| *ft.* | | | | |
|---|---|---|---|---|
| 25 | 6′ | length | | |
| 2 | 7′ | breadth | | |
| 51 | 0′ | | | |
| 14 | 10′ | 6″ | | |
| 65 | 10′ | 6″ | | |
| 3 | 3′ | thickness | | |
| 197 | 7′ | 6″ | | |
| 16 | 5′ | 7″ | 6‴ | |
| 214 | 1′ | 1″ | 6‴ | *Ans.* |

4. Multiply 9*ft.* 2*in.* by 9*ft.* 6*in.* *Ans.* 87*ft.* 1′

5. Multiply 24*ft.* 10*in.* by 6*ft.* 8*in.* *Ans.* 165*ft.* 6′ 8″.

6. Multiply 70*ft.* 9*in.* by 12*ft.* 3*in.* *Ans.* 866*ft.* 8′ 3″.

7. How many cords and cord feet in a pile of wood 24 feet long, 4 feet wide, and 3*ft.* 6*in.* high?

*Ans.* 2 *cords* and 5 *cord feet.*

NOTE. It must be recollected that 16 solid feet make one cord foot § **65**.

**Q. In multiplication how do you set down the multiplier? Where do you begin to multiply? How do you carry from one denomination to another? Repeat the four principles.**

---

## ALLEGATION MEDIAL.

§ **172**. A merchant mixes 8*lb.* of tea worth 75*cts.* per pound, with 16*lb.* worth $1,02 per pound: what is the value of the mixture per pound?

The manner of finding the price of this mixture is called *Allegation Medial.* Hence,

ALLEGATION MEDIAL *teaches the method of finding the price of a mixture when the simples of which it is composed, and their prices, are known.*

In the example above, the simples 8*lb.* and 16*lb.*, and also their prices per pound, 75*cts.* and $1,02, are known.

| | |
|---|---|
| 8*lb.* of tea at 75*cts.* per *lb.* - - - | 6,00 |
| 16*lb.* - - - $1,02 per *lb.* - - - | 16,32 |
| 24 sum of simples. Total cost | $22,32. |

Now if the entire cost of the mixture, which is $22,32, be divided by 24 the number of pounds, or sum of the simples, the quotient 93*cts.* will be the price per pound. Hence, we have the following

OPERATION.

```
24)$22,32(93cts.
    216
    ---
     72
     72
    ---
```

RULE.

*Divide the entire cost of the whole mixture by the sum of the simples: the quotient will be the price of the mixture.*

EXAMPLES.

1. A farmer mixes 30 bushels of wheat worth 5*s* per bushel, with 72 bushels of rye at 3*s* per bushel, and with 60 bushels of barley worth 2*s* per bushel: what is the value of a bushel of the mixture?

| | | |
|---|---|---|
| 30 | bushels of wheat at 5*s* . . | 150*s*. |
| 72 | . . . . rye at 3*s* . . . . | 216*s*. |
| 60 | . . . . barley at 2*s* . . | 120*s*. |
| 162 | | 162)486(3*s*. |
| | | 486 |

*Ans*. 3*s*.

2. A wine merchant mixes 15 gallons of wine at $1 per gallon with 25 gallons of brandy worth 75 cents per gallon: what is the value of a gallon of the compound? *Ans*. 84*cts*.+

3. A grocer mixes 40 gallons of whiskey worth 31*cts*. per gallon with 3 gallons of water, which costs nothing: what is the value of a gallon of the mixture? *Ans*. $28\frac{36}{43}$*cts*.

4. A goldsmith melts together 2*lb*. of gold of 22 carats fine, 6*oz*. of 20 carats fine, and 6*oz*. of 16 carats fine: what is the fineness of the mixture? *Ans*. $20\frac{2}{3}$ *carats*.

5. On a certain day the mercury in the thermometer was observed to average the following heights: from 6 in the morning to 9, 64°; from 9 to 12, 74°; from 12 to 3, 84°; and from 3 to 6, 70°: what was the mean temperature of the day? *Ans*. 73°.

**Q. What is Allegation Medial? How do you find the price of the mixture?**

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## ALLEGATION ALTERNATE.

§ **173**. A farmer would mix oats worth 3*s* per bushel with wheat worth 9*s* per bushel, so that the mixture shall be worth 5*s* per bushel: what proportion must be taken of each sort?

The method of finding how much of each sort must be taken, is called *Allegation Alternate*. Hence, ALLEGATION ALTERNATE *teaches the method of finding what proportion must be taken of several simples, whose prices are known, to form a compound of a given price.*

Allegation Alternate is the reverse of Allegation Medial, and may be proved by it.

For a first example, let us take the one before stated. If oats worth 3*s* per bushel be mixed with wheat worth 9*s*, how much must be taken of each sort that the compound may be worth 5*s* per bushel?

| | | | |
|---|---|---|---|
| 5 | 3 | 4 | Oats. |
| | 9 | 2 | Wheat. |

If the price of the mixture were 6*s*, half the sum of the prices of the simples, it is plain that it would be necessary to take just as much oats as wheat.

But since the price of the mixture is *nearer* to the price of the oats than to that of the wheat, less wheat will be required in the mixture than oats.

Having set down the prices of the simples under each other, and linked them together, we next set 5*s*, the price of the mixture, on the left. We then take the difference between 9 and 5 and place it opposite 3, the price of the oats, and also the difference between 5 and 3 and place it opposite 9, the price of the wheat. The difference standing opposite each kind shows how much of that kind is to be taken. In the present example, the mixture will consist of 4 bushels of oats and 2 of wheat; and any other quantities, bearing the same proportion to each other, such as 8 and 4, 20 and 10, &c., will give a mixture of the same value.

PROOF BY ALLEGATION MEDIAL.

| | |
|---|---|
| 4 bushels of oats at 3*s* . . . . . . | 12*s*. |
| 2 bushels of wheat at 9*s*. . . . . . | 18*s*. |
| 6 | 6)30 |
| | *Ans.* 5*s*. |

Q. What is Allegation Alternate? How do you prove Allegation Alternate?

CASE I.

§ **174**. To find the proportion in which several simples of given prices must be mixed together, that the compound may be worth a given price.

## RULE.

I. *Set down the prices of the simples under each other, in the order of their values, beginning with the lowest.*

II. *Link the least price with the greatest, and the next least with the next greatest, and so on, until the price of each simple which is less than the price of the mixture is linked with one or more that is greater; and every one that is greater with one or more that is less.*

III. *Write the difference between the price of the mixture and that of each of the simples opposite that price with which the particular simple is linked; then the difference standing opposite any one price, or the sum of the differences when there is more than one, will express the quantity to be taken of that price.*

### EXAMPLES.

1. A merchant would mix wines worth 16*s*, 18*s* and 22*s* per gallon in such a way that the mixture be worth 20*s* per gallon: how much must be taken of each sort?

| | | |
|---|---|---|
| 20 | 16 | 2 at 16*s*. |
| | 18 | 2 at 18*s*. |
| | 22 | 4+2=6 at 22*s*. |

*Ans.* 2*gal.* at 16*s*, 2 at 18*s*, and 6 at 22*s*: or any other quantities bearing the proportion of 2, 2 and 6.

2. What proportions of coffee at 16*cts.*, 20*cts.*, and 28*cts.* per lb. must be mixed together so that the compound shall be worth 24*cts.* per lb.?

*Ans.* In the proportion of 4*lb.* at 16*cts.*, 4*lb.* at 20*cts.*, and 12*lb.* at 28*cts.*

3. A goldsmith has gold of 16, of 18, of 23 and of 24 carats fine: what part must be taken of each so that the mixture shall be 21 carats fine?

*Ans.* 3 of 16, 2 of 18, 3 of 23, and 5 of 24.

4. What portion of brandy at 14*s* per gallon, of old Madeira at 24*s* per gallon, of new Madeira at 21*s* per gallon, and of brandy at 10*s* per gallon, must be mixed together so that the mixture shall be worth 18*s* per gallon?

*Ans.* 6*gal.* at 10*s*, 3 at 14*s*, 4 at 21*s*, and 8*gal.* at 24*s*.

### CASE II.

§ 176. When a given quantity of one of the simples is to be taken.

### RULE.

I. *Find the proportional quantities of the simples as in Case* I.

II. *Then say, as the number opposite the simple whose quantity is given, is to the given quantity, so is either proportional quantity to the part of its simple to be taken.*

### EXAMPLES.

1. How much wine at 5*s*, at 5*s* 6*d*, and 6*s* per gallon, must be mixed with 4 gallons at 4*s* per gallon, so that the mixture shall be worth 5*s* 4*d* per gallon?

| | | | |
|---|---|---|---|
| 64 | 48 | 8 | . . simple whose quantity is known. |
| | 60 | 2 | proportional quantities. |
| | 66 | 4 | |
| | 72 | 16 | |

Then 8 : 4 :: 2 : 1
8 : 4 :: 4 : 2
8 : 4 :: 16 : 8

*Ans.* 1*gal.* at 5*s*, 2 at 5*s* 6*d*, and 8 at 6*s*.

### PROOF BY ALLEGATION MEDIAL.

| | | | |
|---|---|---|---|
| 4 *gal.* | at 4*s* per gal. | . . . | 192*d.* |
| 1 | . . . 5*s* | „ . . . | 60 |
| 2 | . . . 5*s* 6*d* | „ . . . | 132 |
| 8 | . . . 6*s* | „ . . . | 576 |
| 15 | | | 15)960(64*d.* price of mixture. |

2. A farmer would mix 14 bushels of wheat, at $1,20 per bushel, with rye at 72*cts.*, barley at 48*cts.*, and oats at 36*cts.*: how much must be taken of each sort to make the mixture worth 64 cents per bushel?

*Ans.* { 14*bu.* of wheat; 8*bu.* of rye; 4*bu.* of barley; and 28*bu.* of oats.

3. There is a mixture made of wheat at 4*s* per bushel, rye at 3*s*, barley at 2*s*, with 12 bushels of oats at 18*d* per

bushel: how much is taken of each sort when the mixture is worth 3*s* 6*d*? *Ans.* { 96*bu.* of wheat; 12*bu.* of rye; 12*bu.* of barley; and 12*bu.* of oats.

4. A distiller would mix 40*gal.* of French brandy at 12*s* per gallon, with English at 7*s* and spirits at 4*s* per gallon: what quantity must be taken of each sort, that the mixture may be afforded at 8*s* per gallon?

*Ans.* { 40*gal.* French; 32*gal.* English; and 32*gal.* of spirits.

CASE III.

**§ 176.** When the quantity of the compound is given as well as the price.

RULE.

I. *Find the proportional quantities as in Case* I.

II. *Then say, as the sum of the proportional quantities, is to the given quantity, so is each proportional quantity, to the part to be taken of each.*

EXAMPLES.

2. A grocer has four sorts of sugar worth 12*d*, 10*d*, 6*d*, and 4*d* per pound; he would make a mixture of 144*lb.* worth 8*d* per pound: what quantity must be taken of each sort?

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 4 | 4 | 12 | : | 144 | :: | 4 | : | 48 |
| | 6 | 2 | 12 | : | 144 | :: | 2 | : | 24 |
| | 10 | 2 | 12 | : | 144 | :: | 2 | : | 24 |
| | 12 | 4 | 12 | : | 144 | :: | 4 | : | 48 |
| Sum of the proportional parts | | 12 | | | | | | | |

*Ans.* { 48*lb.* at 4*d*; 24*lb*, at 6*d*; 24*lb.* at 10*d*; and 48*lb.* at 12*d*.

PROOF BY ALLEGATION MEDIAL.

| | | |
|---|---|---|
| 48*lb.* at 4*d* | . . . . . . . . | 192*d.* |
| 24*lb.* „ 6*d* | . . . . . . . . | 144*d.* |
| 24*lb.* „ 10*d* | . . . . . . . . | 240*d.* |
| 48*lb.* „ 12*d* | . . . . . . . . | 576*d.* |
| 144 | | 144)1152(8*d* |

Hence, the average cost is 8*d*.

2. A grocer having four sorts of tea worth 5*s*, 6*s*, 8*s* and 9*s* per *lb.* wishes a mixture of 87*lb.* worth 7*s* per *lb.*: how much must be taken of each sort?

*Ans.* { 29*lb.* at 5*s*; 14½*lb.* at 6*s*; 14½*lb.* at 8*s*; and 29*lb.* at 9*s*.

3. A vintner has four sorts of wine, viz., white wine at 4*s* per gallon, Flemish at 6*s* per gallon, Malaga at 8*s* per gallon, and Canary at 10*s* per gallon: he would make a mixture of 60 gallons to be worth 5*s* per gallon: what quantity must be taken of each?

*Ans.* { 45*gal.* of white wine; 5*gal.* of Flemish; 5*gal.* of Malaga; and 5*gal.* of Canary.

4. A silver-smith has four sorts of gold, viz; of 24 carats fine, of 22 carats fine, of 20 carats fine, and of 15 carats fine: he would make a mixture of 42*oz.* of 17 carats fine: how much must be taken of each sort?

*Ans.* { 4 of 24; 4 of 22; 4 of 20; and 30 of 15 carats fine.

Q. How do you find the proportional parts when the price only is given? What is the rule when a given quantity of one of the simples is to be taken? What is the rule when the quantity of the compound, as well as the price, is given?

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## INVOLUTION.

§ **177**. If a number be multiplied by itself, the product is called the *second power*, or *square* of that number. Thus $4\times4=16$: the number 16 is the 2nd power or square of 4.

If a number be multiplied by itself, and the product arising be again multiplied by the number, the second product is called the 3rd *power*, or *cube* of the number. Thus $3\times3\times3=27$: the number 27 is the 3rd *power*, or *cube* of 3.

The term *power* designates the product arising from multiplying a number by itself a certain number of times, and the number multiplied is called the *root*.

Thus, in the first example above, 4 is the root, and 16 the square or 2nd power of 4.

In the 2nd example, 3 is the root, and 27 the 3rd power or cube of 3. The first power of a number is the number itself.

Q. If a number be multiplied by itself once, what is the product called? If it be multiplied by itself twice, what is the product called? What does the term power mean? What is the root?

§ **178**. *Involution teaches the method of finding the powers of numbers.*

The number which designates the power to which the root is to be raised, is called the *index* or *exponent* of the power. It is generally written on the right, and a little above the root. Thus $4^2$ expresses the second power of 4, or that 4 is to be multiplied by itself once: hence, $4 = 4\times4=16$.

For the same reason $3^3$ denotes that 3 is to be raised to the 3rd power, or cubed: hence

$3^3=3\times3\times3=27$: we may therefore write,

| | |
|---|---|
| $4=4$ | the 1st power of 4. |
| $4^2=4\times4=16$ | the 2nd power of 4. |
| $4^3=4\times4\times4=64$ | the 3rd power of 4. |
| $4^4=4\times4\times4\times4=256$ | the 4th power of 4. |
| $4^5=4\times4\times4\times4\times4=1024$ | the 5th power of 4. |
| &c. &c. | &c. |

Q. What is Involution? What is the number called which designates the power? Where is it written?

Hence, to raise a number to any power, we have the following

### RULE.

*Multiply the number continually by itself as many times less 1 as there are units in the exponent: the last product will be the power sought.*

### EXAMPLES.

1. What is the 3rd power of 125?
   *Ans.* $125\times125\times125=1953125$.
2. What is the cube of 7? *Ans.* 343.
3. What is the square of 60? *Ans.* 3600.
4. What is the 4th power of 5? *Ans.* 625.

5. What is the 5th power of 9? *Ans.* 59049.
6. What is the cube of 1? *Ans.* 1.
7. What is the square of $\frac{1}{2}$? *Ans.* $\frac{1}{4}$.
8. What is the cube of ,1? *Ans.* ,001.
9. What is the cube of $\frac{3}{8}$? *Ans.* $\frac{27}{512}$.
10. What is the square of ,01? *Ans.* ,0001.
11. What is the square of 2,04? *Ans.* 4,1616.
12. What is the 5th power of 10? *Ans.* 100000.
13. What is the cube of $2\frac{1}{4}$? *Ans.* $11\frac{25}{64}$.

**Q. How do you raise a number to any power?**

---

## EVOLUTION.

§ **179**. We have seen (§ **178**), that Involution teaches how to find the power when the root is given. Evolution is the reverse of Involution: it teaches how to find the root when the power is known. The root is that number which being multiplied by itself a certain number of times will produce the given power.

The square root of a number is that number which being multiplied by itself once will produce the given number.

The cube root of a number is that number which being multiplied by itself twice will produce the given number.

For example, 6 is the square root of 36; because $6\times6=36$; and 3 is the cube root of 27, because $3\times3\times3=27$. The sign $\sqrt{\phantom{x}}$ placed before a number denotes that its square root is to be extracted. Thus, $\sqrt{36}=6$. The sign $\sqrt{\phantom{x}}$ is called the sign of the square root.

When we wish to express that the cube root is to be extracted, we place the figure 3 over the sign of the square root: thus, $\sqrt[3]{8}=2$ and $\sqrt[3]{27}=3$.

Q. What is Evolution? What does it teach? What is the square root of a number? What is the cube root of a number? Make the sign denoting the square root? How do you denote the cube root?

## EXTRACTION OF THE SQUARE ROOT.

§ **180**. To extract the square root of a number is to find a number which, being multiplied by itself once, will produce the given number. Thus $\sqrt{4}=2$; for, $2\times2=4$. And $\sqrt{9}=3$; for, $3\times3=9$.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Roots . . . | 1, | 2, | 3, | 4, | 5, | 6, | 7, | 8, | 9. |
| Squares . . | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81. |

From which we see that the square of either of the significant figures is less than 100, and hence the square root of any number expressed by two figures will be less than 10. It is also evident that there are but nine perfect squares between 1 and 100 among the whole numbers.

Q. What is required when we wish to extract the square root of a number? What is the greatest square of a single figure? Is the square of a single figure always less than 100? Will the square root of two figures be less than 10?

### CASE I.

§ **181**. To extract the square root of a whole number.

### RULE.

I. *Point off the given number into periods of two figures each, counted from the right, by setting a dot over the place of units, another over the place of hundreds, and so on.*

II. *Find the greatest square in the first period on the left, and place its root on the right after the manner of a quotient in division. Subtract the square of the root from the first period, and to the remainder bring down the second period for a dividend.*

III. *Double the root already found and place it on the left for a divisor. Seek how many times the divisor is contained in the dividend, exclusive of the right hand figure, and place the figure in the root and also at the right of the divisor.*

IV. *Multiply the divisor, thus augmented, by the last figure of the root, and subtract the product from the dividend, and to the remainder bring down the next period for a new dividend. But if the product should exceed the dividend, diminish the last figure of the root.*

V. *Double the whole root already found, for a new divisor, and continue the operation as before, until all the periods are brought down.*

EXAMPLES.

1. What is the square root of 263169?

| OPERATION. |
|---|
| 26 31 69(513 |
| 25 |
| 101)131 |
| 101 |
| 1023)3069 |
| 3069 |

We first place a dot over the 9, making the right hand period 69. We then put a dot over the 1 and also over the 6, making three periods.

The greatest perfect square in 26, is 25, the root of which is 5. Placing 5 in the root, subtracting its square from 26, and bringing down the next period 31, we have 131 for a dividend, and by doubling the root we have 10 for a divisor. Now 10 is contained in 13, 1 time. Place 1 both in the root and in the divisor: then multiply 101 by 1; subtract the product and bring down the next period.

We must now double the whole root 51 for a new divisor, or we may take the first divisor after having doubled the last figure 1; then by dividing we obtain 3, the third figure of the root.

NOTE 1. There will be as many figures in the root as there are periods in the given number.

NOTE 2. If the given number has not an exact root, there will be a remainder after all the periods are brought down, in which case ciphers may be annexed, forming new periods, each of which will give one decimal place in the root.

2. What is the square root of 36729?

| |
|---|
| 3 67 29(191,64+. |
| 1 |
| 29)267 |
| 261 |
| 381)629 |
| 381 |
| 3826)24800 |
| 22956 |
| 38324)184400 |
| 153296 |
| 31104 Rem. |

In this example there are two periods of decimals, which give two places of decimals in the root.

3. What is the square root of 106929? *Ans.* 327.
4. What is the square root of 2268741? *Ans.* 1506,23+.
5. What is the square root of 7596796? *Ans.* 2756,22+.
6. What is the square root of 36372961? *Ans.* 6031.
7. What is the square root of 22071204? *Ans.* 4698.

Q. How do you extract the square root of a whole number? How many figures will there be in the root? If the given number has not an exact root, what may be done?

CASE II.

§ 182. To extract the square root of a decimal fraction.

RULE.

I. *Annex one cipher, if necessary, so that the number of decimal places shall be even.*

II. *Point off the decimals into periods of two figures each, by putting a point over the place of hundredths, a second over the place of ten thousandths, &c.: then extract the root as in whole numbers, recollecting that the number of decimal places in the root will be equal to the number of periods in the given decimal.*

EXAMPLES.

1. What is the square root of ,5?

We first annex one cipher to make even decimal places. We then extract the root of the first period, to which we annex ciphers, forming new periods.

OPERATION.

```
      ,50(,707+
       49
  140)100
       000
 1407)10000
       9849
        151 Rem.
```

Note. When there is a decimal and a whole number joined together the same rule will apply.

2. What is the square root of 3271,4207? *Ans.* 57,19+.
3. What is the square root of 4795,25731? *Ans.* 69,247+.

4. What is the square root of 4,372594?

*Ans.* 2,091+.

5. What is the square root of ,00032754?

*Ans.* ,01809+.

6. What is the square root of ,00103041? *Ans.* ,0321.

7. What is the square root of 4,426816? *Ans.* 2,104.

8. What is the square root of 47,692836? *Ans.* 6,906.

Q. How do you extract the square root of a decimal fraction? When there is a decimal and a whole number joined together, will the same rule apply?

CASE III.

§ **183**. To extract the square root of a vulgar fraction.

RULE.

I. *Reduce mixed numbers to improper fractions, and compound fractions to simple ones, and then reduce the fraction to its lowest terms.*

II. *Extract the square root of the numerator and denominator separately, if they have exact roots; but when they have not, reduce the fraction to a decimal and extract the root as in Case II.*

1. What is the square root of $\frac{2304}{5184}$? *Ans.* $\frac{2}{3}$.
2. What is the square root of $\frac{2704}{4225}$? *Ans.* $\frac{4}{5}$.
3. What is the square root of $\frac{9216}{12544}$? *Ans.* $\frac{6}{7}$.
4. What is the square root of $\frac{275}{341}$? *Ans.* ,89802+.
5. What is the square root of $\frac{357}{476}$? *Ans.* ,86602+.
6. What is the square root of $\frac{478}{549}$? *Ans.* ,93309+.

Q. How do you extract the square root of a vulgar fraction?

## EXTRACTION OF THE CUBE ROOT.

§ **184**. To extract the cube root of a number is to find a *second* number which being multiplied into itself twice, shall produce the given number.

Thus, 2 is the cube root of 8; for, $2\times2\times2=8$: and 3 is the cube root of 27; for, $3\times3\times3=27$.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Roots | 1, | 2, | 3, | 4, | 5, | 6, | 7, | 8, | 9. |
| Cubes | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729. |

CASE I.

§ **185**. To extract the cube root of a whole number.

RULE.

I. *Point off the given number into periods of three figures each, by placing a dot over the place of units, a second over the place of thousands, and so on to the left: the left hand period will often contain less than three places of figures.*

II. *Seek the greatest cube in the first period, and set its root on the right after the manner of a quotient in division. Subtract the cube of this figure from the first period, and to the remainder bring down the first figure of the next period, and call the number the dividend.*

III. *Take three times the square of the root just found for a divisor and see how often it is contained in the dividend and place the quotient for a second figure of the root. Then cube the figures of the root thus found, and if their cube be greater than the first two periods of the given number, diminish the last figure, but if it be less, subtract it from the first two periods, and to the remainder bring down the first figure of the next period, for a new dividend.*

IV. *Take three times the square of the whole root for a new divisor, and seek how often it is contained in the new dividend: the quotient will be the third figure of the root. Cube the whole root and subtract the result from the first three periods of the given number, and proceed in a similar way for all the periods.*

EXAMPLES.

1. What is the cube root of 99252847?

$$\dot{9}9\,25\dot{2}\,84\dot{7}(463$$
$$4^3=64$$
$$4^2\times3=48)352 \text{ dividend}$$

First two periods - - - - $\dot{9}9\,25\dot{2}$

$(46)^3=46\times46\times46=$ 97 336

$3\times(46)^2=6348$ ) 19168 2nd dividend

The first three periods - - $\dot{9}9\,25\dot{2}\,84\dot{7}$.

$(463)^3$ $=99\,252\,847.$

*Ans.* 463.

2. What is the cube root of 389017? *Ans.* 73.
3. What is the cube root of 5735339? *Ans.* 179.
4. What is the cube root of 32461759? *Ans.* 319.
5. What is the cube root of 84604519? *Ans.* 439.
6. What is the cube root of 259694072? *Ans.* 638.
7. What is the cube root of 48228544? *Ans.* 364.
8. What is the cube root of 27054036008? *Ans.* 3002.

Q. What is required when we are to extract the cube root of a number? How do you extract the cube root of a whole number?

CASE II.

§ **186.** To extract the cube root of a decimal fraction.

RULE.

*Annex ciphers to the decimal, if necessary, so that it shall consist of* 3, 6, 9, *&c., places. Then put the first point over the place of thousandths, the second over the place of millionths, and so on over every third place to the right; after which extract the root as in whole numbers.*

Note 1. There will be as many decimal places in the root as there are periods in the given number.

Note 2. The same rule applies when the given number is composed of a whole number and a decimal.

Note 3. If in extracting the root of a number there is a remainder, after all the periods have been brought down, periods of ciphers may be annexed by considering them as decimals.

EXAMPLES.

1. What is the cube root of ,157464? *Ans.* ,54.
2. What is the cube root of ,870983875? *Ans.* ,955.
3. What is the cube root of 12,977875? *Ans.* 2,35.
4. What is the cube root of ,751089429. *Ans.* 0,909.
5. What is the cube root of ,353393243. *Ans.* 0,707.
6. What is the cube root of 3,408862625. *Ans.* 1,505.
7. What is the cube root of 27,708101576. *Ans.* 3,026.

Q. How do you extract the cube root of a decimal fraction? How many decimal places will there be in the root? Will the same rule apply when there is a whole number and a decimal? In extracting the root if there is a remainder, what may be done?

CASE III.

§ 187. To extract the cube root of a vulgar fraction.

RULE.

I. *Reduce compound fractions to simple ones, mixed numbers to improper fractions, and then reduce the fraction to its lowest terms.*

II. *Then extract the cube root of the numerator and denominator separately, if they have exact roots; but if either of them has not an exact root, reduce the fraction to a decimal, and extract the root as in the last Case.*

EXAMPLES.

1. What is the cube root of $\frac{250}{686}$? *Ans.* $\frac{5}{7}$.
2. What is the cube root of $12\frac{19}{27}$? *Ans.* $2\frac{1}{3}$.
3. What is the cube root of $31\frac{15}{343}$? *Ans.* $3\frac{1}{7}$.
4. What is the cube root of $\frac{324}{1500}$? *Ans.* $\frac{3}{5}$.
5. What is the cube root of $\frac{4}{7}$? *Ans.* ,829+.
6. What is the cube root of $\frac{5}{9}$? *Ans.* ,822+.
7. What is the cube root of $\frac{2}{3}$? *Ans.* ,873+.

Q. How do you extract the cube root of a vulgar fraction?

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## ARITHMETICAL PROGRESSION.

§ 188. If we take any number, as 2, we can, by the continued addition of any other number, as 3, form a *series* of numbers: thus,

2, 5, 8, 11, 14, 17, 20, 23, &c.,

in which each number is formed by the addition of 3 to the preceding number.

This series of numbers may also be formed by subtracting 3 continually from the larger number: thus,

23, 20, 17, 14, 11, 8, 5, 2.

A series of numbers formed in either way is called an *Arithmetical Series*, or an *Arithmetical Progression*; and the number which is added or subtracted is called the *common difference.*

When the series is formed by the continued addition of the common difference, it is called an *ascending* series; and when it is formed by the subtraction of the common difference, it is called a *descending* series; thus,

2, 5, 8, 11, 14, 17, 20, 23, is an ascending series.
23, 20, 17, 14, 11, 8, 5, 2, is a descending series.

The several numbers are called *terms* of the progression: the first and last terms are called the *extremes*, and the intermediate terms are called the *means.*

Q. How do you form an Arithmetical Series? What is the common difference? What is an ascending series? What a descending series? What are the several numbers called? What are the first and last terms called? What are the intermediate terms called?

§. **189.** In every arithmetical progression there are five things which are considered, any three of which being given or known, the remaining two can be determined. They are,

1st, the first term;
2nd, the last term;
3rd, the common difference;
4th, the number of terms;
5th, the sum of all the terms.

Q. In every Arithmetical Progression how many things are considered? What are they?

§ **190.** By considering the manner in which the ascending progression is formed, we see that the 2nd term is obtained by adding the common difference to the first term; the 3rd, by adding the common difference to the 2nd; the 4th, by adding the common difference to the 3rd, and so on; *the number of additions being* 1 *less than the number of the term found.*

But instead of making the additions, we may multiply the common difference by the number of additions, that is by 1 less than the number of terms, and add the first term to the product.

Hence, we have

### CASE I.

Having given the first term, the common difference, and the number of terms, to find the last term.

### RULE.

*Multiply the common difference by* 1 *less than the number of terms, and to the product add the first term.*

Q. How do you find the last term when the first term and common difference are known?

### EXAMPLES.

1. The first term is 3, the common difference 2, and the number of terms 19: what is the last term?

We multiply the number of terms less 1, by the common difference 2, and then add the first term.

OPERATION.

| | |
|---|---|
| 18 | number of terms less 1. |
| 2 | common difference |
| 36 | |
| 3 | 1st term |
| 39 | last term. |

*Ans.* 39.

2. A man bought 50 yards of cloth for which he was to pay 6 cents for the first yard, 9 cents for the 2nd, 12 cents for the 3d, and so on increasing by the common difference 3: how much did he pay for the last yard? *Ans.* \$1,53.

3. A man puts out \$100 at simple interest, at 7 per cent; at the end of the first year it will have increased to \$107, at the end of the 2nd year to \$114, and so on, increasing \$7 each year: what will be the amount at the end of 16 years? *Ans.* \$205.

4. Twelve persons agree to contribute to a charitable object in the following proportions: the first person is to give \$2, the 2nd \$4, the 3rd \$6, and so on, each giving \$2 more than the one previous: what does the last one give? *Ans.* \$24.

5. The first term is 5, the common difference 12, and the number of terms 15: what is the last term? *Ans.* 173.

§ **191**. Since the last term of an arithmetical progression is equal to the first term added to the product of the common difference by 1 less than the number of terms, it follows, that the difference of the extremes will be equal to this product, and that the common difference will be equal to this product divided by 1 less than the number of terms.

Hence, we have

CASE II.

Having given the two extremes and the number of terms of an arithmetical progression, to find the common difference.

RULE.

*Subtract the less extreme from the greater and divide the remainder by* 1 *less than the number of terms, the quotient will be the common difference.*

Q. How do you find the common difference, when you know the two extremes and number of terms?

EXAMPLES.

1. The extremes are 4 and 104, and the number of terms 26: what is the common difference?

We subtract the less extreme from the greater and divide the difference by one less than the number of terms.

OPERATION.

$$\begin{array}{r} 104 \\ 4 \\ \hline 26-1=25)100(4 \\ 100 \\ \hline \end{array}$$

*Ans.* 4.

2. A man has 8 sons, the youngest is 4 years old and the eldest 32, their ages increase in arithmetical progression: what is the common difference of their ages?

$$32-4=28: \text{ then } 8-1=7)28(4$$

*Ans.* 4.

3. A man is to travel from New York to a certain place in 12 days; to go 3 miles the first day, increasing every day by the same number of miles, so that the last day's journey may be 58 miles: required the daily increase. *Ans.* 5 *miles*

§ **192.** If we take any arithmetical series, as

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19, | &c. |
| 19 | 17 | 15 | 13 | 11 | 9 | 7 | 5 | 3 | by reversing the order of the terms. |
| 22 | 22 | 22 | 22 | 22 | 22 | 22 | 22 | 22 | |

Here we see that the sum of the terms of these two series is equal to 22, the sum of the extremes, multiplied by the number of terms; and consequently, the sum of either series is equal to the sum of the two extremes multiplied by half the number of terms; hence, we have

CASE III.

To find the sum of all the terms of an arithmetical progression.

RULE.

*Add all the extremes together and multiply their sum by half the number of terms, the product will be the sum of the series.*

Q. How do you find the sum of an arithmetical series?

EXAMPLES.

1. The extremes are 2 and 100, and the number of terms 22: what is the sum of the series?

We first add together the two extremes and then multiply by half the number of terms.

OPERATION.

| | |
|---|---|
| 2 | 1st term |
| 100 | last term |
| 102 | sum of extremes |
| 11 | half the number of terms |
| 1122 | sum of series. |

*Ans.* 1122.

2. How many strokes does the hammer of a clock strike in 12 hours? *Ans.* 78.

3. The first term of a series is 2, the common difference 4, and the number of terms 9, what is the last term and sum of the series? *Ans.* last term 34, sum 162.

4. If 100 eggs are placed in a right line, exactly one yard from each other, and the first one yard from a basket: what distance will a man travel who gathers them up singly, and places them in the basket?

*Ans. 5 miles,* 1300 *yards.*

## GEOMETRICAL PROGRESSION.

§ **193.** If we take any number, as 3, and multiply it continually by any other number, as 2, we form a series of numbers, thus,

3 6 12 24 48 96 192, &c, in which each number is formed by multiplying the number before it, by 2.

This series may also be formed by dividing continually the largest number 192 by 2. Thus,

192 96 48 24 12 6 3.

A series formed in either way is called a Geometrical Series, or a Geometrical Progression, and the number by which we continually multiply or divide, is called the *common ratio.*

When the series is formed by multiplying continually by the common ratio, it is called an *ascending series;* and when it is formed by dividing continually by the common ratio, it is called a *descending series.*

Thus,

3 6 12 24 48 96 192 is an ascending series.
192 96 48 24 12 6 2 is a descending series.

The several numbers are called *terms* of the progression.

The first and last terms are called the *extremes*, and the intermediate terms are called the *means.*

Q. How do you form a Geometrical Progression? What is the common ratio? What is an ascending series? What is a descending series? What are the several numbers called? What are the first and last terms called? What are the intermediate terms called?

§ **194.** In every Geometrical, as well as in every Arithmetical Progression, there are five things which are considered, any three of which being given or known, the remaining two can be determined.

They are,

1st the first term,
2nd the last term,
3rd the common ratio,
4th the number of terms,
5th the sum of all the terms.

By considering the manner in which the ascending progression is formed, we see that the second term is obtained by multiplying the first term by the common ratio; the 3rd term by multiplying this product by the common ratio, and so on, the number of multiplications being one less than the number of terms. Thus,

$3=1$ 1st term,
$3\times2=6$ 2nd term,
$3\times2\times2=12$ 3rd term,
$3\times2\times2\times2=24$ 4th term, &c. for the other terms.
But $2\times2=2^2$, $2\times2\times2=2^3$, and $2\times2\times2\times2=2^4$.

Therefore, any term of the progression is equal to the first term multiplied by the ratio raised to a power 1 less than the number of the term.

**Q. In every Geometrical Progression, how many things are considered? What are they?**

CASE I.

Having given the first term, the common ratio, and the number of terms, to find the last term.

RULE.

*Raise the ratio to a power whose exponent is one less than the number of terms, and then multiply the power by the first term, the product will be the last term.*

EXAMPLES.

1. The first term is 3 and the ratio 2; what is the 6th term?

$2\times2\times2\times2\times2=2^5=32$
3 1st term
*Ans.* 96

2. A man purchased 12 pears: he was to pay 1 farthing for the first, 2 farthings for the 2nd, 4 for the 3rd, and so on doubling each time: what did he pay for the last?
*Ans.* £2 2*s.* 8*d.*

3. A gentleman dying left nine sons, and bequeathed his estate in the following manner: to his executors £50;

his youngest son to have twice as much as the executors, and each son to have double the amount of the son next younger: what was the eldest son's portion?

*Ans.* £25600.

4. A man bought 12 yards of cloth, giving 3 cents for the 1st yard, 6 for the 2nd, 12 for the 3rd, &c.: what did he pay for the last yard? *Ans.* $61,44.

CASE II.

§ **195**. Having given the ratio and the two extremes to find the sum of the series.

RULE.

*Subtract the less extreme from the greater, divide the remainder by* 1 *less than the ratio, and to the quotient add the greater extreme: the sum will be the sum of the series.*

Q. How do you find the sum of the series?

EXAMPLES.

1. The first term is 3, the ratio 2, and last term 192: what is the sum of the series?

192—3=189 difference of the extremes,

2−1=1)189(189; then 189+192=381 *Ans.*

2. A gentleman married his daughter on New Year's day, and gave her husband 1*s.* towards her portion, and was to double it on the first day of every month during the year: what was her portion?

*Ans.* £204 15*s.*

3. A man bought 10 bushels of wheat on the condition that he should pay 1 cent for the 1st bushel, 3 for the 2nd, 9 for the 3rd, and so on to the last: what did he pay for the last bushel and for the 10 bushels?

*Ans.* last bushel $196,83, total cost $295,24.

4. A man has 6 children; to the 1st he gives $150, to the 2nd $300, to the 3rd $600, and so on, to each twice as much as the last: how much did the eldest receive and what was the amount received by them all?

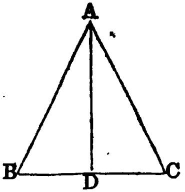
*Ans.* Eldest $4800, total $9450.

# APPENDIX.

## MENSURATION.

§ **196**. A triangle is a figure bounded by three straight lines. Thus, BAC, is a triangle.

The three lines BA, AC, BC, are called *sides:* and the three corners, B, A, and C, are called *angles*. The side BC is called the *base*.




When a line like AD is drawn making the angle ADB equal to the angle ADC, then AD is said to be perpendicular to BC, and AD is called the *altitude* of the triangle. Each triangle BAD or DAC is called a right angled triangle. The side BA or the side AC, opposite the right angle, is called the *hypothenuse.*

*The area or content of a triangle is equal to half the product of its base by its altitude.*

EXAMPLES.

1. The base of a triangle is 40 yards and the perpendicular 20 yards: what is the area?

We first multiply the base by the altitude and the product is square yards, which we divide by 2 for the area.

OPERATION.

$$\begin{array}{r} 40 \\ 20 \\ \hline 2)800 \\ \hline 400 \end{array}$$

*Ans.* 400 *square yards.*

2. In a triangular field the base is 40 chains and the perpendicular 15 chains: how much does it contain? (see § **64**.) *Ans.* 30 *acres.*

3. There is a triangular field of which the base is 35 rods and the perpendicular 26 rods: what is its content? *Ans.* 2*A*. 3*R*. 15*P*.

4. What is the area of a square field of which the sides are each 33,08 chains?

*Ans.* 109*A*. 1*R*. 28*P*+.

5. What is the area of a square piece of land of which the sides are 27 chains? *Ans.* 72*A*. 3*R*. 24*P*.

6. What is the area of a square piece of land of which the sides are 25 rods each? *Ans.* 3*A*. 3*R*. 25*P*.

§ **197**. A rectangle is a four-sided figure like a square, in which the sides are perpendicular to each other, but the adjacent sides are not equal.

*The area or content of a rectangle is equal to the length multiplied by the breadth.*

EXAMPLES.

1. What is the content of a rectangular field the length of which is 40 rods and the breadth 20 rods?

*Ans.* 5 *acres.*

2. What is the content of a field 40 rods square.

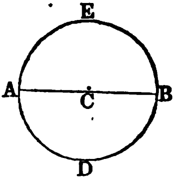
*Ans.* 10 *acres.*

3. What is the content of a rectangular field 15 chains long and 5 chains broad? *Ans.* 7*A*. 2*R*.

4. What is the content of a field 25 chains long by 20 chains broad? *Ans.* 50 *acres.*

5. What is the content of a field 27 chains long and 9 rods broad? *Ans.* 6*A*. 0*R*. 12*P*.

§ **198**. A circle is a portion of a plane bounded by a curved line, every part of which is equally distant from a certain point within, called the centre.




The curved line AEBD is called the *circumference*: the point C the *centre*; the line AB passing through the centre, *a diameter*, and CB the *radius*.

The circumference AEBD is 3,1416 times greater than the diameter AB. Hence, if the diameter is 1, the circumference will be 3,1416. Hence, also, if the diameter is known, the circumference is found by multiplying 3,1416 by the diameter.

EXAMPLES.

1. The diameter of a circle is 4, what is the circumference?

The circumference is found by simply multiplying 3,1416 by the diameter.

OPERATION.

3,1416
4
*Ans.* 12,5664.

2. The diameter of a circle is 93, what is the circumference? *Ans.* 292,1688.

3. The diameter of a circle is 20, what is the circumference? *Ans.* 62,832.

§ **199**. Since the circumference of a circle is 3,1416 times greater than the diameter, it follows that if the circumference is known we may find the diameter by dividing it by 3,1416.

EXAMPLES.

1. What is the diameter of a circle whose circumference is 78,54.

We divide the circumference by 3,1416, the quotient 25 is the diameter.

OPERATION.

3,1416)78,5400(25
62832
157080
157080

2. What is the diameter of a circle whose circumference is 11652,1944? *Ans.* 37,09.

3. What is the diameter of a circle whose circumference is 6850? *Ans.* 2180,41+.

§ **200**. The area or content of a circle is found by *multiplying the square of the diameter by the decimal* ,7854.

EXAMPLES.

1. What is the area of a circle whose diameter is 6?

We first square the diameter, giving 36, which we then multiply by the decimal ,7854: the product is the area of the circle.

OPERATION.

$6^2=36$
$,7854\times 36=28,2744$
*Ans.* 28,2744

2. What is the area of a circle whose diameter is 10? *Ans.* 78,54.

3. What is the area of a circle whose diameter is 7? *Ans.* 38,4846.

4. How many square yards in a circle whose diameter is $3\frac{1}{2}$ feet. *Ans.* 1,069016+.

§ **201**. The surface of a sphere is formed by *multiplying the square of the diameter by the decimal* 3,1416.

EXAMPLES.

1. What is the surface of a sphere whose diameter is 12?

We simply multiply the decimal 3,1416 by the square of the diameter: the product is the surface.

OPERATION.

3,1416
$12^2 = 144$
*Ans.* 452,3904

2. What is the surface of a sphere whose diameter is 7? *Ans.* 153,9384.

3. Required the number of square inches in the surface of a sphere whose diameter is 2 feet or 24 inches? *Ans.* 1809,5616.

4. Required the area of the surface of the earth, its mean diameter being 7918,7 miles? *Ans.* 196996571,722104 *sq. miles.*

§ **202**. To find the solidity of a sphere—*Multiply the surface by the diameter and divide the product by* 6—*the quotient will be the solidity.*

EXAMPLES.

1. What is the solidity of a sphere whose diameter is 12?

We first find the surface by multiplying the square of the diameter by 3,1416. We then multiply the surface by the diameter and divide the product by 6.

OPERATION.

| | |
|---|---|
| | $\overline{12}^2 = 144$ |
| multiply by | 3,1416 |
| surface | =452,3904 |
| diameter | 12 |
| | 6)5428,6848 |
| solidity | =904,7808. |

2. What is the solidity of a sphere whose diameter is 4? *Ans.* 33,5104.

3. What is the solidity of the earth, its mean diameter being 7918,7 miles? *Ans.* 259992792079,860+.

§ **203**. To find the solid content of a prism—*Multiply the area of the base by the altitude, and the product will be the content.*

EXAMPLES.

1. What is the content of a square prism, each side of the square which forms the base being 15, and the altitude of the prism 20 feet?

We first find the area of the square which forms the base, and then multiply by the altitude.

OPERATION.

$\overline{15}^2=225$
20
*Ans.* 4500

2. What is the solid content of a cube each side of which is 24 inches? *Ans.* 13824 *solid in.*

3. How many cubic feet in a block of marble of which the length is 3 feet 2 inches, breadth 2 feet 8 inches, and height or thickness 2 feet 6 inches? *Ans.* $21\frac{1}{9}$ *solid ft.*

4. How many gallons of water, ale measure, will a cistern contain, whose dimensions are the same as in the last example? (See § **67**, Note). *Ans.* $129\frac{17}{47}$*gal.*

5. Required the solidity of a triangular prism whose height is 10 feet, and area of the base 350? *Ans.* 3500.

§ **204**. To find the convex surface of a cylinder—*Multiply the circumference of its base by the altitude.*

EXAMPLES.

1. What is the convex surface of a cylinder, the diameter of whose base is 20 and the altitude 50?

We first multiply the diameter by 3,1416 which gives the circumference of the base. Then multiplying by the altitude, we obtain the convex surface.

OPERATION.

3,1416
20
62,8320
50
*Ans.* 3141,6000

2. Required the convex surface of a cylinder, the circumference of whose base is 6509 and altitude 27?

*Ans.* 175743.

3. Required the surface of a cylinder, the diameter of whose base is 20 and the altitude 20? *Ans.* 1256,64.

§ **205**. To find the solidity of a cylinder—*Multiply the area of the base by the altitude, the product will be the solid content.*

EXAMPLES.

1. Required the solidity of a cylinder of which the altitude is 12 feet, and the diameter of the base 15 feet?

We first find the area of the base, and then multiply by the altitude—the product is the solidity.

OPERATION.

$$15^2=225$$

| | |
|---|---|
| | ,7854 |
| area base | 176,7150 |
| | 12 |
| | 2120,5800 |

2. What is the solidity of a cylinder the diameter of whose base is 20 and the altitude 29? *Ans.* 9110,64.

3. What is the solidity of a cylinder the diameter of whose base is 12, and the altitude 30? *Ans.* 3392,928.

4. What is the solidity of a cylinder, the diameter of whose base is 16 and altitude 9? *Ans.* 1809,5616.

5. What is the solidity of a cylinder, the diameter of whose base is 50 and altitude 15? *Ans.* 29452,5.

§ **206**. To find the solidity of a cone—*Multiply the area of the base by the altitude, and divide the product by* 3.

EXAMPLES.

1. Required the solidity of a cone the diameter of whose base is 5 and the altitude 10?

We first square the diameter and multiply it by ,7854 which gives the area of the base. We next multiply by the altitude, and then divide the product by 3.

OPERATION.

$$5^2=25$$

$$25\times,7854=19,635$$

| |
|---|
| 10 |
| 3)196,35 |
| *Ans.* 65,45 |

2. What is the solidity of a cone the diameter of whose base is 18 and the altitude 27? *Ans.* 2290,2264.

3. What is the solid content of a cone the diameter of whose base is 20 and the altitude 30? *Ans.* 3141,6.

4. What is the solidity of a cone whose altitude is 27 feet, and the diameter of the base 10 feet? *Ans.* 706,86.

5. What is the solidity of a cone whose altitude is 12 feet, and the diameter of its base 15 feet? *Ans.* 706,86.

§ **207**. To find the solidity of a pyramid—*Multiply the area of the base by the altitude and divide the product by* 3.

EXAMPLES.

1. Required the solidity of a pyramid of which the area of the base is 95, and the altitude 15.

We simply multiply the area of the base 95, by the altitude 15, and then divide the product by 3.

OPERATION.

```
     95
     15
    ---
    475
    95
   ----
 3)1425
   ----
Ans. 475
```

2. What is the solidity of a pyramid, the area of whose base is 260 and the altitude 24? *Ans.* 2080.

3. What is the solidity of a pyramid, the area of whose base is 207 and altitude 18? *Ans.* 1242.

4. What is the solidity of a pyramid, the area of whose base is 403 and altitude 30? *Ans.* 4030.

5. What is the solid content of a pyramid, the area of whose base is 270 and altitude 16? *Ans.* 1440.

6. A pyramid has a rectangular base, the sides of which are 25 and 12; the altitude of the pyramid is 36; what is its solid content? *Ans.* 3600.

7. A pyramid with a square base of which each side is 30, has an altitude of 20: what is its solid content? *Ans.* 6000.

## PROMISCUOUS QUESTIONS.

1. A merchant bought 13 packages of goods, for which he paid \$326: what will 39 packages cost at the same rate? *Ans.* \$978.

2. Two merchants, A and B traded together; A put in £320 for 5 months, and B £460 for 3 months; they gained £100: how much should each one receive?
*Ans.* A £53 13*s* $9\frac{270}{298}$*d*, B £46 6*s* $2\frac{28}{298}$*d*.

3. If I buy 1000 Ells Flemish of linen, for £90, what must it be sold for per Ell English, to make £10 by the purchase? *Ans.* 3*s* 4*d*.

4. What number taken from the square of 54 will leave 19 times 46? *Ans.* 2042.

5. If $\frac{5}{6}$ of a gallon of wine cost $\frac{5}{8}$ of a £, what will $\frac{5}{9}$ of a tun cost? *Ans.* £105.

6. If an officer's salary is £48 per annum, how much will he receive in 232 days? *Ans.* £30 10*s* $2\frac{1}{4}$*d*+.

7. If a gentleman spends one day with another, £1 7*s* $10\frac{1}{2}$*d*, and at the end of the year has saved £340, what is his yearly income? *Ans.* £848 14*s* $4\frac{1}{2}$*d*.

8. If 8 cannons expend, in one day, 48 barrels of powder, how much will 24 cannons expend in 22 days?
*Ans.* 3168*bar.*

9. What number is that which being multiplied by $\frac{2}{3}$ will produce $\frac{1}{4}$? *Ans.* $\frac{3}{8}$.

10. A person dying divided his property between his widow and his four sons: to his widow he gave \$1780, and to each of his sons \$1250: he had been $25\frac{1}{2}$ years in business, and had cleared on an average \$126 a year: how much had he when he began business?
*Ans.* \$3567.

11. A besieged garrison consisting of 360 men was provisioned for 6 months, but hearing of no relief at the end of 5 months, dismissed so many of the garrison that the remaining provision lasted 5 months: how many men were sent away? *Ans.* 288.

12. What number added to $11\frac{5}{7}$, will produce $36\frac{337}{616}$?
*Ans.* $24\frac{513}{616}$.

13. What number multiplied by $\frac{3}{7}$, will produce $11\frac{9}{17}$?
*Ans.* $26\frac{46}{51}$.

14. A man had 12 sons, the youngest was 3 years old and the eldest 58, and their ages increased in arithmetical progression: what was the common difference of their ages? *Ans.* 5 *years.*

15. A snail in getting up a pole, 20 feet high, was observed to climb up 8 feet every day, but to descend 4 feet every night: in what time did he reach the top of the pole? *Ans.* 4 *days.*

16. Two persons, A and B are indebted to C; A owes $2173, which is the least debt, and the difference of the debts is $371: what is B's debt? *Ans.* $2544.

17. What is the difference between twice four and forty, and twice forty-four: also between twice five and fifty, and twice fifty-five? *Ans.* 40 and 50.

18. A lady being asked her age, and not wishing to give a direct answer, said, I have 9 children, and three years elapsed between the birth of each of them; the eldest was born when I was 19 years old, and the youngest is now exactly 19: what was her age? *Ans.* 62 *years.*

19. What number added to the 43d part of 4429, will make the sum 240? *Ans.* 137.

20. A man went to sea at 17 years of age; 8 years after he had a son born, who lived 46 years, and died before his father: after which the father lived twice twenty years and died: what was the age of the father?
*Ans.* 111 *years.*

21. A brigade of horse consisting of 384 men, is to be formed into a solid body consisting of 32 men in front; how many ranks will there be? *Ans.* 12.

22. A room 30 feet long and 18 feet wide is to be covered with painted cloth $\frac{3}{4}$ of a yard in width: how many yards will cover it? *Ans.* 80.

23. A, B and C trade together and gain $120, which is to be shared according to each one's stock; A put in

\$140, B \$300, and C \$160; what is each man's share? *Ans.* A's \$28, B's \$60, and C's 32.

24 There is a stone which measures 4 feet 6 inches long, 2 feet 9 inches broad, and 5 feet 4 inches deep: how many solid feet does it contain? *Ans.* 66 *feet.*

25. Two men depart from the same place and travel in different directions; one goes 7 miles and the other 11 miles per day: how far will they be apart at the end of the 12th day? *Ans.* 216 *miles.*

26. How many planks 15 feet long and 15 inches wide, will floor a barn $60\frac{1}{2}$ feet long and $33\frac{1}{2}$ feet wide?

*Ans.* $108\frac{7}{75}$.

27. A person owned $\frac{3}{5}$ of a mine, and sold $\frac{3}{4}$ of his interest for \$1710: what was the value of the entire mine?

*Ans.* \$3800.

28. The swiftest velocity of a cannon ball, is about 2000 feet in a second of time. In what time, at that rate, would it be in moving from the earth to the sun, admitting the distance to be 95 millions of miles, and the year to contain 365 days 6 hours. *Ans.* $7\frac{12457}{13149}$ *years.*

29. The slow or parade step is 70 paces per minute, at 28 inches each pace: how fast is that per hour?

*Ans.* $1\frac{113}{132}$ *miles.*

30. A wall of 700 yards in length was to be built in 29 days. Twelve men were employed on it for 11 days, and only completed 220 yards. How many men must be added to complete the wall in the required time? *Ans.* 4.

31. How far will 500 millions of guineas reach, when laid down in a straight line touching one another, supposing each guinea to be an inch in diameter?

*Ans.* 7891*mi.* 728*yd.* 2*ft.* 8*in.*

32. A gentleman whose annual income is £1500, spends 20 guineas a week: does he save or run in debt, and how much? *Ans.* He saves £408 *per annum.*

33. A person bought 160 oranges at 2 for a penny, and 180 more at 3 for a penny; after which he sold them out at the rate of 5 for 2 pence: did he make or lose, and how much? *Ans.* He lost 4 *pence.*

34. My factor sends me word that he has bought goods to the value of £500 13*s* 6*d* upon my account: what will his commission come to at 3½ per cent.?

*Ans.* £17 10*s* 5½*d*+.

35. If a quantity of provisions serves 1500 men 12 weeks, at the rate of 20 ounces a day for each man; how many men will the same provisions maintain for 20 weeks, at the rate of 8 ounces a day for each man?

*Ans.* 2250 *men.*

36. A younger brother received $8400, which was just $\frac{7}{8}$ of his elder brother's fortune: what was the father worth? *Ans.* $19200.

37. If 20 men can perform a piece of work in 12 days, how many men will accomplish three times as much in one-fifth of the time? *Ans.* 300.

38. Suppose that I have $\frac{3}{16}$ of a ship worth $1200; what part have I left after selling $\frac{2}{5}$ of $\frac{4}{9}$ of my share, and what is it worth? *Ans.* $\frac{37}{240}$ left, worth $986,66+.

39. What number is that which being multiplied by $\frac{2}{3}$ of $\frac{4}{5}$ of 1½, the product will be 1? *Ans.* 1¼.

40. What number is that which being multiplied by three thousandths, the product will be 2637?

*Ans.* 879000.

41. What length must be cut off a board 8½ inches broad to contain a square foot, or as much as 12 inches in length and 12 in breadth? *Ans.* $16\frac{16}{17}$ *inches.*

42. A man exchanged 70 bushels of rye, at $0,92 per bushel, for 40 bushels of wheat, at $1,37½ per bushel, and received the balance in oats, at $0,40 per bushel: how many bushels of oats did he receive? *Ans.* 23½*bu.*

43. My horse and saddle together are worth $132, and the horse is worth 10 times as much as the saddle: what is the value of the horse. *Ans.* $120.

44. Four persons traded together on a capital of $6000, of which A put in ½, B put in ¼, C put in $\frac{1}{6}$, and D the rest; at the end of 4 years they had gained $4728. what was each one's share of the gain?

*Ans.* A's $2364. B's $1182. C's $ 788. D's $ 394.

45. A farmer being asked how many sheep he had, answered, that he had them in five fields, in the 1st he had $\frac{1}{4}$ of his flock, in the 2nd $\frac{1}{6}$, in the 3rd $\frac{1}{8}$, in the 4th $\frac{1}{12}$, and in the 5th 450: how many had he? *Ans.* 1200.

46. The circumference of the earth is 360 degrees, and each degree is 69$\frac{1}{2}$ miles, how long would a man be in travelling round it, who travelled at the rate of 20 miles a day, the year being reckoned at 365 days 6 hours?

*Ans.* 3 *years* 155$\frac{1}{4}$ *days.*

47. How many bricks 8 inches long and 4 inches wide, will pave a yard that is 100 feet by 50 feet; also a yard that is 50 feet square? *Ans.* 22500;—*2nd yard* 11250.

48. Sound travels about 1142 feet in a second. Now if the flash of a cannon be seen at the moment it is fired, and the report heard 45 seconds after, what distance would the observer be from the gun?

*Ans.* 9*mi.* 5*fur.* 34*rd*+.

49. Two persons depart from the same place, one travels 32, and the other 36 miles a day: if they travel in the same direction, how far will they be apart at the end of 19 days, and how far if they travel in contrary directions?

*Ans.* { 76 *miles* same direction.
1292 *miles* opposite directions. }

50. In a certain orchard, $\frac{1}{2}$ of the trees bear apples, $\frac{1}{4}$ of them bear peaches, $\frac{1}{6}$ of them plums, 120 of them cherries, and 80 of them pears: how many trees are there in the orchard? *Ans.* 2400.

51. A person being asked the time, said, the time past noon is equal to $\frac{1}{5}$ of the time past midnight: what was the hour? *Ans.* 3 *o'clock.*

52. A circular fish pond is 865 feet in diameter: what is its circumference, and what is its area?

*Ans.* { circumference 2717,484*ft.*
area 587655,915*sq. ft.* }

53. How many stones 2 feet long, 1 foot wide, and 6 inches thick, will build a wall 12 yards long, 2 yards high, and 4 feet thick? *Ans.* 864.

54. A well is to be stoned, of which the diameter is 6 feet 6 inches, the thickness of the wall is to be 1 foot 6 inches, leaving the diameter of the well within the stones, 3 feet 6 inches. If the well is 40 feet deep, how many feet of stone will be required? *Ans.* 942,48*ft.*

55. A reservoir of water has two cocks to supply it. The first would fill it in 40 minutes, and the second in 50 It has likewise a discharging cock by which it may be emptied when full in 25 minutes. Now if all the cocks are opened at once and the water runs uniformly as we have supposed, how long before the cistern will be filled?

*Ans.* 3*hr.* 20*m.*

56. A ship has a leak by which it would fill and sink in 15 hours, but by means of a pump it could be emptied, if full, in 16 hours. Now, if the pump is worked from the time the leak begins, how long before the ship will sink?

It will fill $\frac{1}{15}$ in an hour; they pump out $\frac{1}{16}$, hence, the water gains $\frac{1}{15}-\frac{1}{16}=\frac{1}{240}$ of the the ship per hour.

*Ans.* 240 *hours.*

57. How many planks 15 feet long and 15 inches wide, will floor a barn which is 60½ feet long and 33½ wide?

*Ans.* $108\frac{7}{15}$.

58. A person dying worth $5460 left his wife with child, to whom he bequeathed, that if she had a son she should have one third of his estate, and the rest go to the son; but if she had a daughter, then the daughter to have one third and the rest go to the mother. Now, it so happened that she had a son and daughter both; how must the estate be divided to fulfil the father's instructions?

*Ans. Daughter* $780, *Son* $3120, *Wife* $1560.

59. A cistern containing 60 gallons of water has three unequal cocks for discharging it; the largest will empty it in 1 hour, the second in two hours, and the third in three: in what time will the cistern be emptied if they all run together? *Ans.* $32\frac{8}{11}$*min.*

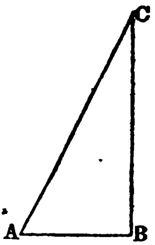
60. A house is 40 feet from the ground to the eaves, and it is required to find the length of a ladder which will reach the eaves, supposing the foot of the ladder cannot be placed nearer to the house than 30 feet.

It is demonstrated in Geometry that in every right angled triangle, such as BAC, the square of the hypothenuse AC is equal to the sum of the squares of the other sides, AB and BC. That is,

$$AC^2 = AB^2 + BC^2.$$

If then we extract the square root, we have

$$AC = \sqrt{AB^2 + BC^2}.$$




When, therefore, the sides AB, BC are known, we can find the side AC, by first squaring AB and BC, taking the sum and extracting the square root.

Thus, in the example above, we square each of the sides, take the sum, which is 2500, the square root of which is 50. Hence, 50 is the length of the required ladder.

$$40^2 = 1600$$
$$30^2 = \phantom{0}900$$
$$2500$$
$$\sqrt{2500} = 50$$

61. If a house is 50 feet deep, and the upright which supports the ridge pole is 12 feet high, what will be the length of the rafters? *Ans.* 27,7*ft.*+.

62. When it is 12 o'clock at New York, what is the hour at London, New York being 75° of Longitude west of London?

Since the circumference of the earth is supposed to be divided into 360 degrees (see § **70**), and since the sun apparently passes through these 360° every twenty-four hours, it follows that in a single hour it will pass through one twenty-fourth of 360° or 15°. Hence there are

15° of motion in 1 hour of time,
1° of motion in 4 minutes,
1′ of motion in 4 seconds.

If two places, therefore, have different longitudes they will have different times, and the difference of time will be one hour for every 15° of longitude, or 4 minutes for each degree, and 4 seconds for each minute. It must be observed that the place which is most *easterly* will have the time first, because the sun travels from east to west.

To return then to our question, the difference of longitude between London and New York being 75°, the difference of time will be found in minutes by multiplying 75° by 4, giving 300 minutes, or 5 hours. Now since New York is west of London, the time will be *later* in New York: that is, when it is twelve o'clock at New York, it will be five, P. M. in London; or when it is 12 at London, it will be 7, A. M. at New York.

OPERATION.

$$\begin{array}{r} 75° \\ 4 \\ \hline 60)300 \\ \hline \textit{Ans. } 5 \textit{ hours.} \end{array}$$

63. Boston is 6° 40′ east longitude from the city of Washington: when it is 6 o'clock P. M. at Washington what is the hour at Boston.

The 6 degrees being multiplied by 4 gives 24 minutes of time, and the 40′ minutes being multiplied by 4 gives 160 seconds, or 2 minutes 40 seconds. The sum is 26′ 40″, and since Boston is east of Washington the time is later at Boston.

OPERATION.

$$\begin{array}{r} 6 \times 4 = 24' \\ 40 \times 4 = 160'' = \;\; 2'\ 40'' \\ \hline 26'\ 40'' \\ \hline \textit{Ans. } 26'\ 40''\ \textit{past } 6. \end{array}$$

64. The difference of longitude of two places is 85° 20′; what is the difference of time?

*Ans.* 5*hr.* 41*m.* 20*sec.*

65. A traveller leaves New Haven at 8 o'clock on Monday morning, and walks towards Albany at the rate of 3 miles an hour; another traveller sets out from Albany at 4 o'clock on the same evening and walks towards New Haven, at the rate of 4 miles an hour; now supposing the distance to be 130 miles, whereabout on the road will they meet? *Ans.* $69\frac{5}{7}$ *miles from New Haven.*

66. What is the least number that can be divided by 1, 2, 3, 4, 5, 6, 7, 8, and 9, without a remainder?

*Ans.* 2520.

67. A thief is escaping from an officer. He has 40 miles the start, and travels at the rate of 5 miles an hour, the officer in pursuit travels at the rate of 7 miles an hour: how far must he travel before he overtakes the thief? *Ans.* He travels 20 *hours*, and 140 *miles.*

68. A can do a piece of work alone in 10 days, and B in 13 days: in what time can they do it if they work together? *Ans.* $5\frac{15}{23}$.

69. The accounts of a certain school are as follows: viz, $\frac{1}{16}$ of the boys learn geometry, $\frac{3}{8}$ learn grammar, $\frac{3}{10}$ learn arithmetic, $\frac{3}{20}$ learn to write, and 9 learn to read: what is the number in each branch?

*Ans.* { 5 learn geometry, 30 grammar, 24 arithmetic, 12 writing, and 9 reading.

70. If $120 be divided among three persons, A, B, and C, so that when A has $3, B shall have 5 and C 7: how much will each receive? *Ans.* A $24, B $40, and C $56.

71. The head diameter of a cask is 20 inches and the bung diameter 26 inches: how many wine gallons does it contain, and how many beer gallons?

The mean diameter of a cask is found by adding to the head diameter, two-thirds of the difference between the bung and head diameters, or if the staves are not much curved, by adding six-tenths. This reduces the cask to a cylinder. Then, to find the solidity, we multiply the square of the mean diameter by the decimal ,7854 and the product by the length;—this will give the solid content in cubic inches. Then if we divide by 231 we have the content in wine gallons (see § **66** Note), or if divide by 282 we have the content in beer gallons (see § **67** Note.)

For wine measure we multiply the length by the square of the mean diameter, then by the decimal ,7854, and divide by 231.

OPERATION.

$$l \times d^2 \times \frac{,7854}{231} = l \times d^2 ,0034$$

If then, we divide the decimal ,7854 by 231, the quotient carried to four places of decimals is ,0034, and this decimal multiplied by the square of the mean diameter and by the length of the cask, will give the content in wine gallons.

For similar reasons, the content is found in beer gallons by multiplying together the length, the square of the mean diameter, and the decimal, ,0028.

OPERATION.

$$l \times d^2 \times \frac{,7854}{282} = = l \times d^2 ,0028$$

Hence for guaging or measuring casks, we have the following

### RULE.

*Multiply the length by the square of the mean diameter, then multiply by* 34 *for wine, and by* 28 *for beer measure, and point off in the product four decimal places. The product will then express gallons, and the decimals of a gallon.*

72. How many wine gallons in a cask, whose bung diameter is 36 inches, head diameter 30 inches, and length 50 inches.

We first find the difference of the diameters, of which we take two thirds and add to the head diameter. We then multiply the square of the mean diameter, the length and 34 together, and point off four decimal places in the product.

OPERATION.

$36-30=6$
$\frac{2}{3}$ of $6=4$
$30+4=34$
$34^2=1156$
$1156\times 50\times 34=$
$=196{,}52$ *gal.*

73. What is the number of beer gallons in the last example? *Ans.* 161,84.

74. How many wine, and how many beer gallons in a cask whose length is 36 inches, bung diameter 35 inches, and head diameter 30 inches? *Ans.* { 136 *wine gal.* 112 *beer gal.*

75. A stationer sold quills at 11*s* a thousand, by which he cleared $\frac{3}{8}$ of the money; but they growing scarce he raised the price to 13*s* 6*d* a thousand: what did he clear at the last price, on each £100 laid out?

*Ans.* £96 7*s* $3\frac{3}{11}$*d.*

76. A water tub holds 147 gallons; the pipe usually brings in 14 gallons in 9 minutes: the tap discharges, at a medium, 40 gallons in 31 minutes. Now, supposing these to be left open, and the water to be turned on at 2 o'clock in the morning; a servant at 5 shuts the tap, and is solicitous to know in what time the tub will be filled in case the water continues to flow.

*Ans. the tub will be full at* 3 *min.* $48\frac{1}{2}\frac{4}{1}\frac{4}{7}$*sec. after* 6.

# FORMS RELATING TO BUSINESS.

## FORMS OF ORDERS.

MESSRS. M. JAMES & Co.

Please pay John Thompson, or order, five hundred dollars, and place the same to my account.

PETER WORTHY.

*New York, June* 1, 1833.

---

MR. JOSEPH RICH,

Please pay the bearer sixty-one dollars and twenty cents, in goods from your store, and charge the same to the account of your

Obedient Servant,

JOHN PARSONS.

*New York, July* 1, 1837.

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## FORMS OF RECEIPTS.

### *Receipt for Money on Account.*

Received, New York, June 2nd, 1837, of John Ward, sixty dollars on account.

$60,00 J. P. FAY.

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### *Receipt for Money on a Note.*

Received New York, June 5, 1837, of Leonard Walsh, six hundred and forty dollars, on his note for one thousand dollars, dated New York, January 1, 1837.

$640,00. J. N. WEEKS,

## FORMS OF NOTES.

*Negotiable Note.*

No. 1.
$25,50. New York, May 1, 1837.

For value received I promise to pay on demand, to Abel Bond, or order, twenty-five dollars and fifty cents.

REUBEN HOLMES.

---

*Note Payable to Bearer.*

No. 2.
$875,39. New York, May 2, 1837.

For value received I promise to pay, six months after date, to John Johns, or bearer, eight hundred and seventy-five dollars and thirty-nine cents.

PIERCE PENNY.

---

*Note by two Persons.*

No. 3.
$659,27. New York, June 2, 1837.

For value received, we, jointly and severally, promise to pay to Richard Ricks, or order, on demand, six hundred and fifty-nine dollars and twenty-seven cents.

ENOS ALLAN.
JOHN ALLAN.

---

*Note Payable at a Bank.*

No. 4.
$20,25. New York, May 7, 1837.

Sixty days after date, I promise to pay John Anderson, or order, at the Bank of the United States, twenty dollars and twenty-five cents, for value received.

JESSE STOKES.

## *Remarks relating to Notes.*

1. The person who signs a note, is called the *drawer* or *maker* of the note: thus, Reuben Holmes is the drawer of note *No.* 1.

2. The person who has the rightful possession of a note, is called the *holder* of the note.

3. A note is said to be *negotiable* when it is made payable to A B, or order, (See No. 1). Now if Abel Bond to whom this note is made payable, writes his name on the back of it, he is said to *endorse* the note, and he is called the endorser; and when the note becomes due, the holder must first demand payment of the maker, Reuben Holmes, and if he declines paying it, the holder may then require payment of Abel Bond, the endorser.

4. If the note is made payable to A B, or bearer, then the drawer alone is responsible, and he must pay to any person who holds the note.

5. The time at which a note is to be paid should always be named, but if no time is specified, the drawer must pay when required to do so, and the note will draw interest after the payment is demanded.

6. When a note, payable at a future day, becomes due, it will draw interest, though no mention is made of interest.

7. In each of the States there is a *rate* of interest established by law, which is called the legal interest, and when no rate is specified, the note will always draw legal interest. If a rate *higher* than legal interest be taken, the drawer, in most of the States, is not bound to pay the note.

8. If two persons jointly and severally give their note, (See No. 3) it may be collected of either of them.

9. The words "For value received," should be expressed in every note.

10. When a note is given, payable on a fixed day, and in a specific article, as in wheat or rye, payment must be offered at the specified time, and if it is not, the holder can demand the value in money.

## A BOND FOR ONE PERSON, WITH A CONDITION.

KNOW ALL MEN BY THESE PRESENTS, THAT *I James Wilson of the City of Hartford and State of Connecticut am* held and firmly bound unto *John Pickens of the Town of Waterbury, County of New Haven and State of Connecticut,* in the sum *of Eighty dollars* lawful money of the United States of America, to be paid to the said *John Pickens his* executors, administrators, or assigns: for which payment well and truly to be made *I* bind *myself, my* heirs, executors, and administrators, firmly by these presents. Sealed with *my* Seal. Dated this *Ninth* day of *March* one thousand eight hundred and *thirty-eight.*

THE CONDITION of the above obligation is such, that if the above bounden *James Wilson, his* heirs, executors, or administrators, shall well and truly pay or cause to be paid, unto the above named *John Pickens, his* executors, administrators, or assigns, the just and full sum of

*Here insert the condition.*

then the above obligation to be void, otherwise to remain in full force and virtue.

Sealed and delivered in the presence of

*John Frost,*
*Joseph Wiggins.* } *James Wilson.* L.S.

NOTE. The part in Italic to be filled up according to circumstance.

If there is no condition to the bond then all to be omitted after the words "THE CONDITION, &c."

## A BOND FOR TWO PERSONS, WITH A CONDITION.

KNOW ALL MEN BY THESE PRESENTS, THAT, *We*, *James Wilson and Thomas Ash of the City of Hartford and State of Connecticut, are* held and firmly bound unto *John Pickens of the Town of Waterbury, County of New Haven and State of Connecticut,* in the sum *of Eighty dollars* lawful money of the United States of America, to be paid to the said *John Pickens, his* executors, or assigns: for which payment well and truly to be made *We* bind *ourselves, our* heirs, executors, and administrators, firmly by these presents. Sealed with *our* Seal. Dated the *Ninth* day of *March* one thousand eight hundred and *thirty-eight.*

THE CONDITION of the above obligation is such, that if the above bounden *James Wilson and Thomas Ash their* heirs, executors, or administrators, shall well and truly pay or cause to be paid, unto the above named *John Pickens his* executors, administrators, or assigns, the just and full sum of

*Here insert the condition.*

then the above obligation to be void, otherwise to remain in full force and virtue.

Sealed and delivered in
the presence of

| | |
|---|---|
| *John Frost,* | *James Wilson,* L.S. |
| *Joseph Wiggins.* | *Thomas Ash.* L.S. |

NOTE. The part in Italic to be filled up according to circumstance.

If there is no condition to the bond, then all to be omitted after the words "THE CONDITION, &c."

# A PRACTICAL SYSTEM OF BOOK-KEEPING.

PERSONS transacting businesss find it necessary to write down the articles bought or sold, together with their prices and the names of the persons with whom the bargains are made.

BOOK-KEEPING is the method of recording such transactions in a regular manner. It is divided into two kinds, called Single Entry and Double Entry. The method by Single Entry is the most simple, and answers for all common business. This method we will explain.

Book-Keeping by Single Entry requires two books, a Day-Book and a Leger; and when cash sales are extensive, an additional book is necessary, which is called a Cash Book.

## DAY-BOOK.

This book should contain a full history of the business transactions, in the precise order in which they may have occurred.

The transfer of an account from the Day-Book to the Leger, is called *posting* the account.

Each page of the Day-Book should be ruled with two columns on the right hand of the page, one for dollars, and one for cents, and one column on the left hand for entering the page of the Leger on which the account may be posted.

The Day-Book should begin with the name of the owner, and his place of residence; and then should follow a full account of the transactions in business in the exact order in which they may have taken plaec,

The name of the person, or customer, is first written with the term *Dr.* or *Cr.* opposite, according as he becomes a debtor or creditor by the transaction.

Generally, the person who receives is Debtor, and the person who parts with his property is the Creditor.

Thus, if I sell goods to A B, on credit, he becomes my debtor to the amount of the goods, and the goods should be specified particularly in making the charge.

If I buy goods on credit, of C D, I enter C D Cr. to the amount of the goods, taking care to specify the goods in the charge.

If I pay money for, or on account of another person, he becomes Dr. to me for the amount paid.

The Day-Book and Leger are generally designated, Day-Book A, Day-Book B, Leger A, Leger B, &c.: for when one book, in the course of business, is filled with charges a new one is taken.

## DAY-BOOK A.

Page 1. *Edward P. Nixon, New York, June* 1, 1837.

| Folio Leger | *New York, June* 1*st*. 1837. | | $ | cts |
|---|---|---|---|---|
| | George Wilson, - - - - - Dr. | | | |
| ×1. | To 11 cwt. of sugar at $9 per cwt. | $99,00 | | |
| | To 66lb. of coffee at 20 cts. pr. lb. | 13,20 | 112 | 20 |
| | Henry Jones, - - - - - - Dr. | | | |
| ×1. | To balance of former account, | $159,10 | | |
| | To 5 gals. of molasses at 32 cts. per gal. - - - - - - - - - | 1,60 | 160 | 70 |
| | *2nd.* | | | |
| | Charles Patch, - - - - - Dr. | | | |
| ×1. | To Cash, - - - - - - - - - | $327,09 | | |
| | To one hogshead of molasses, - | 124,02 | 451 | 11 |
| | John Dodson, - - - - - - Dr. | | | |
| ×1. | To 10 pieces of cloth at $4,50 per piece. - - - - - - - | $45,00 | | |
| | To 15 yards of calico at 25 cts. per yard - - - - - - - - | 3,75 | 48 | 75 |
| | *3rd.* | | | |
| | William Wells, - - - - - Dr. | | | |
| ×1. | To 400lb. of beef at $8,25 per 100lb. - - - - - - - - - - | $33,00 | | |
| | To 6000lb. of cheese at $10 cwt. | 600,00 | 633 | 00 |
| | Henry Jones, - - - - - - Cr. | | | |
| ×1 | By Cash, - - - - - - - - - - - | | 160 | 70 |

Page 2. *New York, June 5th,* 1837.

| Folio Leger | | | $ | cts |
|---|---|---|---|---|
| | George Wilson, - - - - - | Cr. | | |
| ×1. | By Cash, - - - - - - - - - - | $100 | | |
| | By his note of date for - - - - - | 12,20 | 112 | 20 |
| | William Wells, - - - - - | Cr. | | |
| ×1. | By Cash, - - - - - - - - - - - | | 633 | 00 |
| | *6th.* | | | |
| | John McNeill, - - - - - - | Dr. | | |
| ×1. | To Cash - - - - - - - - - - | $275,10 | | |
| | To one horse, - - - - - - - | 125,00 | | |
| | To 85lb. of butter at 20 cts. per lb. | 17,00 | 417 | 10 |
| | Daniel Judson, - - - - - - | Dr. | | |
| | To 3 *hhd.* of molasses at $20 each, | $60,00 | | |
| ×1. | To 3 *bar.* of salted shad at $8 per barrel - - - - - - - - - - | 24,00 | | |
| | To 4 kegs of raisins at $2 per kg. | 8,00 | 92 | 00 |
| | Charles Patch, - - - - - - | Cr. | | |
| ×1. | By Cash, - - - - - - - - - - | $400,00 | | |
| | By his note of this date, due Aug. 1, 1837, - - - - - - - - | 51,11 | 451 | 11 |
| | *8th.* | | | |
| ×1. | Daniel Judson, - - - - - - | Cr. | | |
| | By 116*lb.* of beef at 8 cts. per *lb.* | $9,28 | | |
| | By 50 *bu.* of oats at 37 cts. per bushel | 18,50 | 27 | 78 |
| | Jared Newton, - - - - - - | Dr. | | |
| ×1. | To 1 piece of linen 36 yards, - - | $42,50 | | |
| | To 3 *yds.* of broadcloth at 4,50 per *yd.* | 13,50 | | |
| | To 46*lb.* of nails at 6 cts. - - - | 2,76 | 58 | 76 |
| | *10th.* | | | |
| ×1. | Jared Newton, - - - - - - | Cr. | | |
| | By Cash, - - - - - - - - - - | $37,50 | | |
| | By do. - - - - - - - - - - - | 21,26 | 58 | 76 |
| | John Dodson, - - - - - - | Cr. | | |
| ×1. | By Cash, - - - - - - - - - - | $24,50 | | |
| | By 20*lb.* of butter at 18 cts. per *lb.* | 3,60 | | |
| | By his note of this date, on demand, | 20,65 | 48 | 75 |
| ×1. | John McNeill, - - - - - | Cr. | | |
| | By Cash, - - - - - - - - - - - | | 100 | 00 |

# LEGER.

The Leger is a book into which is collected, in a condensed form, all the scattered accounts from the Day-Book.

Two columns should be ruled on the right of each page of the Leger, one for dollars and one for cents; there should also be two columns on the left, one to insert the date of the transaction, and the other for inserting the page of the Day-Book from which the account is transferred.

Two pages of the Leger, facing each other, are generally used in stating an account, though sometimes a page is divided into two equal parts, in which case each part may be considered as forming a separate page. The name of the person with whom the account is stated should be written in large letters at the top of the left-hand page.

The Debits are entered on the left-hand page, and the Credits on the other page directly opposite. The difference between the debits and credits, is always entered under the least sum, when the account is closed, and is called the *balance*, as in the account of John McNeill.

At the top of the left-hand column, we enter the year, under which, we enter the day of the month on which the transaction took place; and in the adjoining column, we enter the page of the Day-Book from which the account is transferred.

When there are several articles charged in the Day-Book, we need not specify them all, but may enter them in the Leger under the general name of "Sundries." Having posted the account, we enter the page of the Leger to which it has been transferred, in the left hand column of the Day-Book and opposite the account, and make a mark × to catch the eye and show that the account is posted.

We begin posting with the account of George Wilson, who stands charged on the Day-Book with $112,20. Having posted the first charge, we run the eye carefully through the Day-Book, to see if there are any other charges against him, and find that in page 2 he is credited by 100 dollars cash, and a note for $12,20. These items we enter in the Leger, on the credit side, and as the debits and credits are equal, his account is balanced. We proceed in the same way to post all the accounts entered in the Day-Book. No erasures should ever be made in the account books. When an error is discovered, if it be in favor of the customer he should be charged with the amount, and if against him, he should be credited with the amount. In posting the account of Jared Newton, a mistake was made against him of $21,26, which was rectified by crediting him with the amount.

When a charge is entered on the wrong side of the book, as when a person is charged with that for which he ought to have been credited, *twice* the amount must be entered on the other side of the book to make the account right.

Every Leger should have an Index, where the names of all persons, who have accounts in the Leger, should be arranged in alphabetical order.

When a Leger is filled, all the accounts are balanced, and when we transfer the balances to a new Leger, we charge "To balance from Leger A, page"—.

## INDEX TO LEGER.

| | *Folio* | | *Folio* | | *Folio* |
|---|---|---|---|---|---|
| D. | | M. | | P. | |
| Dodson, John | 1 | McNeill, John | 1 | Patch, Charles | 1 |
| J. | | N. | | W. | |
| Jones, Henry | 1 | Newton, Jared | 1 | Wells, William | 1 |
| Judson, Daniel | 1 | | | Wilson, George | 1 |

## LEGER A.



*Edward P. Nixon, New York, June* 1, 1837.

| 1837. | D. B. Folio | | | $ | cts |
|---|---|---|---|---|---|
| | | Dr. George Wilson, - | | | |
| June 1. | 1. | To Sundries, - - - | $112,20 | 112 | 20 |
| June 1. | 1. | Dr. Henry Jones, - - | | | |
| | | To Sundries, - - - | $160,70 | 160 | 70 |
| June 2. | 1. | Dr. Charles Patch, - | | | |
| | | To Cash, - - - - - | $327,09 | | |
| | | To one hogshead of molasses, | 124,02 | 451 | 11 |
| June 2. | 1. | Dr. John Dodson, - - | | | |
| | | To Sundries, - - - - - | $48,75 | 48 | 75 |
| June 3. | 1. | Dr. William Wells, - - | | | |
| | | To Sundries, - - - | 633,00 | 633 | 00 |
| June 6. | 1. | Dr. John McNeill, - - | | | |
| | | To Sundries, - - - | $417,10 | 417 | 10 |
| June 6. | 1. | Dr. Daniel Judson, - - | | | |
| | | To Sundries, - - - | $92,00 | 92 | 00 |
| June 8. | 1. | Dr. Jared Newton, - - | | | |
| | | To Sundries, - - - | $58,76 | 58 | 76 |